In the case of $2 \text{ ft} + 3 \text{ ft} = 5 \text{ ft}$; 2, 3, and 5 are counting, numerating, and ft names the type of thing that is being counted, the denomination. In the case of $2 + 3 = 5$; 2, 3, and 5 are counting again, and $\$ is the denomination. In both cases the denomination does not change through the addition. ft + ft = ft, and $\$ + \$ = \$. In fractions the top number is counting, i.e. numerating; so it is called the numerator; and the bottom number is identifying the type of piece that is being counted, i.e. denominating—literally, naming—so it is called the denominator. As with the ft and $\$ examples above, $\frac{2}{7} + \frac{3}{7} = \frac{5}{7}$.

Now try adding 3 dimes to 2 quarters. 2 quarters + 3 dimes = what? You could say it is 5 coins, or you can say it is 80 cents. Either way, you have changed the denomination to a common denomination. 2 quarters + 3 dimes = 2 coins + 3 coins = 5 coins, or 2 quarters + 3 dimes = 50 cents + 30 cents = 80 cents. Subtraction is similar.

2 quarters – 3 dimes, taking three of one thing from two of another thing, makes no sense, but if someone exchanges your two quarters for the monetary equivalent, five dimes, the problem becomes 5 dimes – 3 dimes = 2 dimes. Again, the denomination was changed to something that does make sense.

If you have a fraction that has a denominator that you do not want you can change that denominator to any multiple of itself without changing the number that the fraction represents. You just multiply both the numerator and denominator by the same number which amounts to multiplying the fraction by one, and multiplication by one does not
change a number. For example, suppose you had the fraction \( \frac{1}{4} \), but you preferred a denominator of 8. Just multiply \( \frac{1}{4} \) by 1 in the form of \( \frac{2}{2} \).

\[
\frac{1}{4} = \frac{1}{4} \times 1 = \frac{1}{4} \times \frac{2}{2} = \frac{2}{8}
\]

So why would you want to do this? Suppose you wanted to add \( \frac{1}{4} \) and \( \frac{3}{8} \). \( \frac{1}{4} + \frac{3}{8} \) as written does not immediately make sense. It says add three of something to one of something else. Before the addition can be executed you must convert the addends to a common form, i.e. a common denominator.

\[
\frac{1}{4} + \frac{3}{8} = \frac{1}{4} \times \frac{2}{2} + \frac{3}{8} = \frac{2}{8} + \frac{3}{8} = \frac{5}{8}
\]

To execute addition or subtraction with fractions the fractions must have a common denominator, a common name. Then you keep that common name and add or subtract the numerators. The work is simplest if you use the smallest common name, i.e. the least common denominator, the LCD. Given any collection of numbers there is always a number that they all divide. That number is a multiple of each, a common multiple. One such number can be found by multiplying all of the numbers in the collection together. Consider 2, 3, 4, and 6. They all go into \( 2 \times 3 \times 4 \times 6 = 144 \). However, they also all go into 12. Wouldn’t you rather use 12 instead of 144? 12 is the smallest number that is divisible by 2, 3, 4, and 6. 6 is the smallest positive number that is divisible by 6. 2 and 3 go into 6, but 4 does not. The next number that 6 divides is 12, and 4 divides 12, so 12 is the smallest number that is divisible by all. 12 is the least common multiple of 2, 3, 4, and 6. Using 12, a problem like \( \frac{1}{2} + \frac{2}{3} + \frac{3}{4} - \frac{5}{6} \) becomes:

\[
\frac{1}{2} + \frac{2}{3} + \frac{3}{4} - \frac{5}{6} = \frac{1}{2} \times \frac{6}{6} + \frac{2}{3} \times \frac{4}{4} + \frac{3}{4} \times \frac{3}{3} - \frac{5}{6} \times \frac{2}{2}
\]

\[
= \frac{6}{12} + \frac{6}{12} + \frac{9}{12} - \frac{10}{12} = \frac{6 + 6 + 9 - 10}{12} = \frac{13}{12}
\]

Using 144, it becomes:

\[
\frac{1}{2} + \frac{2}{3} + \frac{3}{4} - \frac{5}{6} = \frac{1}{2} \times \frac{72}{72} + \frac{2}{3} \times \frac{48}{48} + \frac{3}{4} \times \frac{36}{36} - \frac{5}{6} \times \frac{24}{24}
\]

\[
= \frac{72}{144} + \frac{96}{144} + \frac{108}{144} - \frac{120}{144} = \frac{72 + 96 + 108 - 120}{144} = \frac{156}{144} = \frac{13}{12} \times \frac{12}{12}
\]

\[
= \frac{13}{12} \times 1 = \frac{13}{12}
\]
You could not leave $\frac{156}{144}$ as your answer. It is reducible, and we always reduce our answers to simplest form. Wasn’t 12 much better than 144? Always use the least common multiple of the denominators, the least common denominator, the LCD.

**Examples:**

\[
\frac{2}{3} - \frac{2}{9} + \frac{3}{4} - \frac{5}{8}
\]

This has LCD = 72. (72 is the smallest number that both 8 and 9 divide, and 4 and 3 divide 8 and 9 respectively, so they, also, divide 72.) \(\frac{72}{3} = 24, \frac{72}{9} = 8, \frac{72}{4} = 18,\) and \(\frac{72}{8} = 9\). So:

\[
\frac{2}{3} - \frac{2}{9} + \frac{3}{4} - \frac{5}{8} = \frac{2}{3} \times \frac{24}{24} - \frac{2}{9} \times \frac{8}{8} + \frac{3}{4} \times \frac{18}{18} - \frac{5}{8} \times \frac{9}{9}
\]

\[
= \frac{48}{72} - \frac{16}{72} + \frac{54}{72} - \frac{45}{72} = \frac{48-16+54-45}{72} = \frac{41}{72}
\]

\[
\frac{5}{3} - \frac{1}{6} - \frac{3}{16} - \frac{5}{8}
\]

This has LCD = 48. \(\frac{48}{3} = 16, \frac{48}{6} = 8, \frac{48}{16} = 3,\) and \(\frac{48}{8} = 6\). So:

\[
\frac{5}{3} - \frac{1}{6} - \frac{3}{16} - \frac{5}{8} = \frac{5}{3} \times \frac{16}{16} - \frac{1}{6} \times \frac{8}{8} - \frac{3}{16} \times \frac{3}{3} - \frac{5}{8} \times \frac{6}{6}
\]

\[
= \frac{80}{48} - \frac{8}{48} - \frac{9}{48} - \frac{30}{48} = \frac{80-8-9-30}{48} = \frac{33}{48} = \frac{3 \times 11}{3 \times 16} = \frac{11}{16}
\]
Example 1: \[ \frac{2}{3} - \frac{2}{9} + \frac{3}{4} - \frac{5}{8} \]

This has LCD = 72. (72 is the smallest number that both 8 and 9 divide, and 4 and 3 divide 8 and 9 respectively, so they, also, divide 72.) \[ \frac{72}{3} = 24, \frac{72}{9} = 8, \frac{72}{4} = 18, \text{ and } \frac{72}{8} = 9. \] So:

\[ \frac{2}{3} - \frac{2}{9} + \frac{3}{4} - \frac{5}{8} = \frac{2 \times 24}{24} - \frac{2}{9} \times \frac{8}{8} + \frac{3 \times 18}{18} - \frac{5}{8} \times \frac{9}{9} \]

\[ = \frac{48 - 16 + 54 - 45}{72} = \frac{41}{72} \]

Example 2: \[ \frac{5}{3} - \frac{1}{6} - \frac{3}{16} - \frac{5}{8} \]

This has LCD = 48. \[ \frac{48}{3} = 16, \frac{48}{6} = 8, \frac{48}{16} = 3, \text{ and } \frac{48}{8} = 6. \] So:

\[ \frac{5}{3} - \frac{1}{6} - \frac{3}{16} - \frac{5}{8} = \frac{5 \times 16}{16} - \frac{1 \times 8}{8} - \frac{3 \times 3}{3} - \frac{5 \times 6}{6} \]

\[ = \frac{80 - 8 - 9 - 30}{48} = \frac{33}{48} = \frac{3 \times 11}{3 \times 16} = \frac{11}{16} \]

Exercises:

1. \[ \frac{7}{3} - \frac{4}{9} + \frac{3}{8} - \frac{5}{4} \]

Again, the LCD = 72. Again, \( \frac{72}{3} = 24, \frac{72}{9} = 8, \frac{72}{8} = 9, \text{ and } \frac{72}{4} = 18. \) Fill in the blanks.

\[ \frac{7}{3} - \frac{4}{9} + \frac{3}{8} = \frac{7 \times 3}{3} - \frac{4 \times 9}{9} + \frac{3 \times 4}{4} - \frac{5 \times 4}{4} \]
\[
\frac{1}{72} - \frac{1}{72} + \frac{1}{72} - \frac{1}{72} = \frac{1}{72} - \frac{1}{72} - \frac{1}{72} = \frac{73}{72}
\]

Ans: \(\frac{73}{72}\)

2. \(\frac{5}{3} - \frac{1}{6} - \frac{7}{16} + \frac{5}{8}\)

Again, the LCD = 48. Again, \(48/3=16\), \(48/6=8\), \(48/16=3\), and \(48/8=6\). Fill in the blanks.

\[
\frac{5}{3} - \frac{1}{6} - \frac{7}{16} + \frac{5}{8} = \frac{5}{3} \times - \frac{1}{6} \times - \frac{7}{16} \times + \frac{5}{8} \times \]

\[
= \frac{48}{48} - \frac{48}{48} - \frac{48}{48} + \frac{48}{48} = \frac{27}{16}
\]

Ans: \(\frac{27}{16}\)

Simplify the following completely.

3. \(\frac{2}{3} - \frac{7}{24}\)

LCD=\(\frac{24}{24}\)

Ans: \(\frac{3}{24}\)

4. \(\frac{5}{7} - \frac{1}{3} + \frac{2}{21}\)

LCD=\(\frac{21}{21}\)

Ans: \(\frac{3}{21}\)
5. \( \frac{5}{12} + \frac{1}{15} - \frac{7}{24} \) 
   LCD=________
   Ans:_______

6. \( \frac{7}{4} - \frac{3}{8} + \frac{11}{12} \)
   LCD=________
   Ans:_______

7. \( \frac{2}{3} - \frac{4}{25} + \frac{7}{15} + \frac{4}{5} \)
   LCD=________
   Ans:_______

8. \( \frac{5}{3} - \frac{1}{36} - \frac{7}{18} + \frac{5}{24} \)
   LCD=________
   Ans:_______
9. \( \frac{5}{4} - \frac{1}{25} - \frac{7}{20} + \frac{3}{10} \) 

\( \text{LCD=} \) 

\( \text{Ans=} \) 

10. \( \frac{5}{7} - \frac{1}{4} + \frac{7}{3} + \frac{5}{12} \) 

\( \text{LCD=} \) 

\( \text{Ans=} \) 

11. \( \frac{5}{14} + \frac{1}{2} - \frac{3}{7} + \frac{5}{49} \) 

\( \text{LCD=} \) 

\( \text{Ans=} \) 

12. \( \frac{5}{9} - \frac{1}{5} - \frac{2}{15} + \frac{5}{3} \) 

\( \text{LCD=} \) 

\( \text{Ans=} \)
Adding and Subtracting Fractions Exercise Set #2

Example 1: \( \frac{2}{3} - \frac{2}{9} + \frac{3}{4} - \frac{5}{8} \)

This has LCD = 72. (72 is the smallest number that both 8 and 9 divide, and 4 and 3 divide 8 and 9 respectively, so they, also, divide 72.) \( 72/3 = 24, 72/9 = 8, 72/4 = 18, \) and \( 72/8 = 9. \) So:

\[
\frac{2}{3} - \frac{2}{9} + \frac{3}{4} - \frac{5}{8} = \frac{2}{3} \times \frac{24}{24} - \frac{2}{9} \times \frac{8}{8} + \frac{3}{4} \times \frac{18}{18} - \frac{5}{8} \times \frac{9}{9}
\]

\[
= \frac{48}{72} - \frac{16}{72} + \frac{54}{72} - \frac{45}{72} = \frac{48-16+54-45}{72} = \frac{41}{72}
\]

Example 2: \( \frac{5}{3} - \frac{1}{6} - \frac{3}{16} - \frac{5}{8} \)

This has LCD = 48. \( 48/3 = 16, 48/6 = 8, 48/16 = 3, \) and \( 48/8 = 6. \) So:

\[
\frac{5}{3} - \frac{1}{6} - \frac{3}{16} - \frac{5}{8} = \frac{5}{3} \times \frac{16}{16} - \frac{1}{6} \times \frac{8}{8} - \frac{3}{16} \times \frac{3}{3} - \frac{5}{8} \times \frac{6}{6}
\]

\[
= \frac{80}{48} - \frac{8}{48} - \frac{9}{48} - \frac{30}{48} = \frac{80-8-9-30}{48} = \frac{33}{48} = \frac{3 \times 11}{3 \times 16} = \frac{11}{16}
\]

Exercises:

1. \( \frac{4}{3} + \frac{4}{9} + \frac{3}{8} - \frac{5}{4} \)

Again, the LCD = 72. Again, \( 72/3=24, 72/9=8, 72/8=9, \) and \( 72/4=18. \) Fill in the blanks.

\[
\frac{4}{3} + \frac{4}{9} + \frac{3}{8} - \frac{5}{4} = \frac{4}{3} \times \frac{3}{3} + \frac{4}{9} \times \frac{9}{9} + \frac{3}{8} \times \frac{8}{8} - \frac{5}{4} \times \frac{4}{4}
\]
\[
\begin{align*}
\frac{72}{72} + \frac{72}{72} + \frac{72}{72} - \frac{72}{72} &= \frac{65}{72} \\
\text{Ans: } &
\end{align*}
\]

2. \[
\frac{5}{3} - \frac{7}{24} + \frac{7}{16} + \frac{5}{8}
\]
Again, the LCD = 48. Again, 48/3=16, 48/24=2, 48/16=3, and 48/8=6. Fill in the blanks.
\[
\frac{5}{3} - \frac{7}{24} + \frac{7}{16} + \frac{5}{8} = \frac{5}{3} \times \frac{48}{48} - \frac{7}{24} \times \frac{48}{48} + \frac{7}{16} \times \frac{48}{48} + \frac{5}{8} \times \frac{48}{48}
\]
\[
= \frac{39}{16}
\]
\[
\text{Ans: } \frac{39}{16}
\]

Simplify the following completely.

3. \[
\frac{7}{48} + \frac{7}{16}
\]
\[
\text{LCD=} \frac{115}{48}
\]
\[
\text{Ans: } \frac{115}{48}
\]

4. \[
\frac{1}{3} - \frac{2}{21} + \frac{5}{2}
\]
\[
\text{LCD=} \frac{115}{42}
\]
\[
\text{Ans: } \frac{115}{42}
\]
5. \( \frac{5}{9} + \frac{1}{5} - \frac{7}{25} \)  
   \( \text{LCD=} \) \________
   \( \text{Ans=} \) \________

6. \( \frac{7}{4} + \frac{5}{6} - \frac{11}{12} + \frac{5}{9} \)  
   \( \text{LCD=} \) \________
   \( \text{Ans=} \) \________

7. \( \frac{5}{9} + \frac{1}{6} - \frac{7}{27} + \frac{5}{18} \)  
   \( \text{LCD=} \) \________
   \( \text{Ans=} \) \________

8. \( \frac{5}{7} - \frac{1}{3} - \frac{2}{21} + \frac{5}{2} \)  
   \( \text{LCD=} \) \________
   \( \text{Ans=} \) \________
9. \[ \frac{8}{3} - \frac{5}{9} + \frac{4}{45} + \frac{3}{25} \]

\[ \text{LCD=} \]

\[ \text{Ans:} \]

10. \[ \frac{5}{9} + \frac{5}{6} - \frac{7}{16} - \frac{5}{48} \]

\[ \text{LCD=} \]

\[ \text{Ans:} \]

11. \[ \frac{5}{4} - \frac{2}{5} - \frac{3}{25} + \frac{1}{50} \]

\[ \text{LCD=} \]

\[ \text{Ans:} \]

12. \[ \frac{5}{12} + \frac{1}{5} - \frac{8}{15} + \frac{5}{48} \]

\[ \text{LCD=} \]

\[ \text{Ans:} \]
Adding and Subtracting Fractions Exercise Set #3

Example 1: \( \frac{2}{3} - \frac{2}{9} + \frac{3}{4} - \frac{5}{8} \)

This has LCD = 72. (72 is the smallest number that both 8 and 9 divide, and 4 and 3 divide 8 and 9 respectively, so they, also, divide 72.) 72/3 = 24, 72/9 = 8, 72/4 = 18, and 72/8 = 9. So:

\[
\frac{2}{3} - \frac{2}{9} + \frac{3}{4} - \frac{5}{8} = \frac{2}{3} \times \frac{24}{24} - \frac{2}{9} \times \frac{8}{8} + \frac{3}{4} \times \frac{18}{18} - \frac{5}{8} \times \frac{9}{9} \\
= \frac{48}{72} - \frac{16}{72} + \frac{54}{72} - \frac{45}{72} = \frac{48-16+54-45}{72} = \frac{41}{72}
\]

Example 2: \( \frac{5}{3} - \frac{1}{6} - \frac{3}{16} - \frac{5}{8} \)

This has LCD = 48. 48/3 = 16, 48/6 = 8, 48/16 = 3, and 48/8 = 6. So:

\[
\frac{5}{3} - \frac{1}{6} - \frac{3}{16} - \frac{5}{8} = \frac{5}{3} \times \frac{16}{16} - \frac{1}{6} \times \frac{8}{8} - \frac{3}{16} \times \frac{3}{3} - \frac{5}{8} \times \frac{6}{6} \\
= \frac{80}{48} - \frac{8}{48} - \frac{9}{48} - \frac{30}{48} = \frac{80-8-9-30}{48} = \frac{33}{48} = \frac{3 \times 11}{3 \times 16} = \frac{11}{16}
\]

Exercises:

1. \( \frac{7}{9} - \frac{5}{36} + \frac{3}{8} + \frac{5}{3} \)

Again, the LCD = 72. Again, 72/9=8, 72/36=2, 72/8=9, and 72/3=24. Fill in the blanks.

\[
\frac{7}{9} - \frac{5}{36} + \frac{3}{8} + \frac{5}{3} = \frac{7}{9} \times \frac{9}{9} - \frac{5}{36} \times \frac{36}{36} + \frac{3}{8} \times \frac{8}{8} + \frac{5}{3} \times \frac{3}{3}
\]
The LCD = 75. Again, 75/3=25, 75/25=3, 75/15=5, and 75/5=15. Fill in the blanks.

\[
\frac{2}{3} + \frac{4}{25} - \frac{7}{15} + \frac{4}{5} = \frac{2}{3} \times \frac{25}{25} + \frac{4}{25} \times \frac{3}{3} - \frac{7}{15} \times \frac{15}{15} + \frac{4}{5} \times \frac{5}{5}
\]

\[
= \frac{75}{75} + \frac{75}{75} - \frac{75}{75} + \frac{75}{75} = \frac{29}{25}
\]

Ans: _________

Simplify the following completely.

3. \[\frac{5}{6} - \frac{5}{24}\]

LCD=_______

Ans:_______

4. \[\frac{1}{3} - \frac{2}{15} + \frac{3}{2}\]

LCD=_______

Ans:_______
5. \( \frac{5}{6} - \frac{7}{27} + \frac{7}{18} \) \[ \text{LCD=} \] \[ \text{Ans=} \]

6. \( \frac{7}{5} + \frac{2}{7} - \frac{3}{14} \) \[ \text{LCD=} \] \[ \text{Ans=} \]

7. \( \frac{7}{4} - \frac{1}{6} - \frac{11}{12} + \frac{5}{9} \) \[ \text{LCD=} \] \[ \text{Ans=} \]

8. \( \frac{7}{5} - \frac{2}{7} - \frac{11}{14} + \frac{5}{2} \) \[ \text{LCD=} \] \[ \text{Ans=} \]
9. \[\frac{5}{3} - \frac{4}{9} - \frac{5}{36} - \frac{5}{8}\] \[\text{LCD}=________\]

Ans:_______

10. \[\frac{5}{3} - \frac{1}{15} - \frac{7}{16} - \frac{5}{8}\] \[\text{LCD}=________\]

Ans:_______

11. \[\frac{5}{2} - \frac{1}{5} - \frac{7}{10} + \frac{5}{7}\] \[\text{LCD}=________\]

Ans:_______

12. \[\frac{4}{3} - \frac{1}{6} - \frac{7}{15} + \frac{5}{7}\] \[\text{LCD}=________\]

Ans:_______