Quick Review 12.1

1. Check the point \((-2, 5)\) in both equations.

<table>
<thead>
<tr>
<th>First Equation</th>
<th>Second Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(4x + 3y = 7)</td>
<td>(5x + 2y = 1)</td>
</tr>
<tr>
<td>(4(-2) + 3(5) = 7)</td>
<td>(5(-2) + 2(5) = 1)</td>
</tr>
<tr>
<td>(-8 + 15 = 7)</td>
<td>(-10 + 10 = 1)</td>
</tr>
<tr>
<td>(7 = 7)</td>
<td>(0 \neq 1)</td>
</tr>
</tbody>
</table>

The point \((-2, 5)\) is not a solution to the system of linear equations.

2. Let \(x = 5\) in either equation and solve for \(y\).

\[
\begin{align*}
2x + 3y &= 4 \\
2(5) + 3y &= 4 \\
10 + 3y &= 4 \\
3y &= -6 \\
y &= -2
\end{align*}
\]

3. \[
\begin{align*}
\frac{x}{2} - \frac{y}{3} &= 1 \\
6\left(\frac{x}{2} - \frac{y}{3}\right) &= 6(1) \\
\frac{2}{1}x - \frac{2}{1}y &= 6 \\
3x - 3y &= 6
\end{align*}
\]

4. **Step 1**

Solve the second equation for \(x\).

\[
\begin{align*}
3x - 4y &= 8 \\
x - 2y &= 2
\end{align*}
\]

Second equation: \(x = 2y + 2\)

**Step 2**

Substitute \(2y + 2\) for \(x\) in the first equation and then solve for \(y\).

\[
\begin{align*}
3x - 4y &= 8 \\
3(2y + 2) - 4y &= 8 \\
6y + 6 - 4y &= 8 \\
2y &= 2 \\
y &= 1
\end{align*}
\]

**Step 3**

Back-substitute 1 for \(y\) in the equation obtained in step 1.

\[
\begin{align*}
x &= 2(1) + 2 \\
x &= 4 \\
\text{Solution: } (4, 1)
\end{align*}
\]

5. To eliminate \(y\), multiply both sides of the first equation by 4, then multiply the second equation by 5 and add the two equations. Solve this equation for \(x\).

\[
\begin{align*}
2x + 5y &= 1 \\
8x + 20y &= 4 \\
3x - 4y &= 13 \\
15x - 20y &= 65 \\
23x &= 69 \\
x &= 3
\end{align*}
\]

Back substitute 3 for \(x\) in the first equation to find \(y\).

\[
\begin{align*}
2x + 5y &= 1 \\
2(3) + 5y &= 1 \\
6 + 5y &= 1 \\
5y &= -5 \\
y &= -1 \\
\text{Solution: } (3, -1)
\end{align*}
\]
Exercises 12.1

2. \[
\begin{bmatrix}
5 & -1 & 3 \\
4 & 3 & 29
\end{bmatrix}
\]

4. \[
\begin{bmatrix}
2 & 7 & 3 \\
0 & -3 & 3
\end{bmatrix}
\]

6. \[
\begin{bmatrix}
1 & 0 & -4 \\
0 & 1 & 11
\end{bmatrix}
\]

8. \[
\begin{align*}
5x - y &= 0 \\
3x + 2y &= 13
\end{align*}
\]

10. \[
\begin{align*}
2x + 3y &= 5 \\
6x - 4y &= 2
\end{align*}
\]

12. \[
\begin{align*}
x &= \frac{2}{3} \\
y &= -\frac{4}{5}
\end{align*}
\]

14. \[
\begin{bmatrix}
-6 & 3 & 4 \\
1 & 2 & 1
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 2 & 1 \\
-6 & 3 & 4
\end{bmatrix}
\]

16. \[
\begin{bmatrix}
1 & -1 & 1 \\
2 & -3 & -2
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -1 & 1 \\
0 & -1 & -4
\end{bmatrix}
\]

18. \[
\begin{bmatrix}
2 & 3 & 1 \\
3 & -3 & -21
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & \frac{3}{2} & \frac{1}{2} \\
3 & -3 & -21
\end{bmatrix}
\]

20. \[
\begin{bmatrix}
1 & -4 & 10 \\
0 & 1 & -2
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 2 \\
0 & 1 & -2
\end{bmatrix}
\]

22. Answer: (4, 6)

24. The last row of the matrix implies 0 = -2 which is a contradiction. Thus there is no solution.

26. Since the last row of the matrix gives us an identity, we have a dependent system of equations with an infinite number of solutions. Solving for \(x\) in the first equation we obtain: \(x = 3 + 2y\). Thus the general solution of the system is \((3 + 2y, y)\). For \(y = 0, y = 2, \text{ and } y = -1,\) we obtain the following particular solutions: \((3, 0), (7, 2), \text{ and } (1, -1)\).

28. \[
\begin{bmatrix}
2 & 5 & 9 \\
3 & 2 & 8
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 5 & 9 \\
3 & 2 & 8
\end{bmatrix}
\]

30. \[
\begin{bmatrix}
1 & 3 & 8 \\
2 & -3 & 7
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 3 & 8 \\
0 & -9 & -9
\end{bmatrix}
\]

32. \[
\begin{bmatrix}
1 & 6 & 2 \\
0 & 3 & 1
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 6 & 2 \\
0 & 1 & \frac{1}{3}
\end{bmatrix}
\]

34. \[
\begin{bmatrix}
1 & 3 & 2 \\
0 & 1 & \frac{1}{3}
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 1 \\
0 & 1 & \frac{1}{3}
\end{bmatrix}
\]

36. \[
\begin{bmatrix}
1 & -2 & 9 \\
3 & 4 & 7
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -2 & 9 \\
0 & 10 & -20
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -2 & 9 \\
0 & 1 & -2
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 5 \\
0 & 1 & -2
\end{bmatrix}
\text{: Answer: (5, -2)}
\]

38. \[
\begin{bmatrix}
1 & 2 & 7 \\
4 & 3 & 3
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 2 & 7 \\
0 & 5 & -25
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 3 \\
0 & 1 & 5
\end{bmatrix}
\text{: Answer: (-3, 5)}
\]
Chapter 12: A preview of College Algebra

40. \[
\begin{bmatrix}
2 & -3 & 17 \\
4 & 1 & 13
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -3 & 17 \\
4 & 1 & 13
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -3 & 17 \\
0 & 7 & -21
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -3 & 17 \\
0 & 1 & -3
\end{bmatrix}
\]
\[
\eta = \frac{1}{3} \cdot \eta_1 \rightarrow
\begin{bmatrix}
1 & 0 & 4 \\
0 & 1 & -3
\end{bmatrix}; \text{ Answer: } (4, -3)
\]

42. \[
\begin{bmatrix}
8 & 3 & -39 \\
7 & -2 & -11
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 3 & 8 \\
7 & -2 & -11
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 3 & 8 \\
0 & 37 & 185
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 3 & -39 \\
0 & 8 & 8
\end{bmatrix}
\]
\[
\eta = \frac{1}{3} \cdot \eta_1 \rightarrow
\begin{bmatrix}
1 & 0 & -3 \\
0 & 1 & -5
\end{bmatrix}; \text{ Answer: } (-3, -5)
\]

44. \[
\begin{bmatrix}
3 & 4 & 0 \\
2 & -3 & 16
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 4 & 3 \\
2 & -3 & 16
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 4 & 0 \\
0 & -17 & 16
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 4 & 0 \\
0 & 1 & 48
\end{bmatrix}
\]
\[
\eta = \frac{1}{3} \cdot \eta_1 \rightarrow
\begin{bmatrix}
1 & 0 & 64 \\
0 & 1 & 48
\end{bmatrix}; \text{ Answer: } \left(\frac{64}{17}, \frac{48}{17}\right)
\]

46. \[
\begin{bmatrix}
5 & -1 & 7 \\
2 & 4 & 5
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -1 & 7 \\
2 & 4 & 5
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -1 & 7 \\
0 & 22 & 11
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -1 & 7 \\
0 & 1 & 12
\end{bmatrix}
\]
\[
\eta = \frac{1}{5} \cdot \eta_1 \rightarrow
\begin{bmatrix}
1 & 0 & 3 \\
0 & 1 & 2
\end{bmatrix}; \text{ Answer: } \left(\frac{3}{5}, \frac{1}{2}\right)
\]

48. \[
\begin{bmatrix}
4 & 3 & 2 \\
16 & 12 & 7
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 3 & 1 \\
16 & 12 & 7
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 3 & 1 \\
0 & 0 & -1
\end{bmatrix}
\]
The last row is a contradiction. Thus there is no solution.

50. \[
\begin{bmatrix}
3 & -6 & 12 \\
4 & -8 & 16
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -2 & 4 \\
4 & -8 & 16
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -2 & 4 \\
0 & 0 & 0
\end{bmatrix}
\]
Since the last row of the matrix gives us an identity, we have a dependent system of equations with an infinite number of solutions. Solving for \(x\) in the first equation we obtain: \(x = 4 + 2y\). Thus the general solution of the system is \((4 + 2y, y)\).

52. Let \(x\) and \(y\) be the two numbers. If their sum is 260 and one number is three times the other, then to find the numbers we solve the following system.
\[
\begin{align*}
x + y &= 260 \\
x - 3y &= 0
\end{align*}
\]
\[
\begin{bmatrix}
1 & 1 & 260 \\
1 & -3 & 0
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 1 & 260 \\
0 & -4 & -260
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 260 \\
0 & 1 & 65
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 195 \\
0 & 1 & 65
\end{bmatrix}
\]
Thus the two numbers are 195 and 65.
54. Let \( x \) and \( y \) be the two angles. If they are supplementary and one angle is \( 74^\circ \) larger than the other, then to find the angles we solve the following system.

\[
\begin{align*}
\begin{array}{c}
\begin{cases}
 x + y = 180 \\
x - y = 74
\end{cases}
\end{array}
\end{align*}
\]

Thus the two angles are \( 127^\circ \) and \( 53^\circ \).

56. Let \( x \) and \( y \) be the rates of the bicyclists. To find these values, solve the following system.

\[
\begin{align*}
\begin{array}{c}
\begin{cases}
 x - y = 5 \\
2x + 2y = 130
\end{cases}
\end{array}
\end{align*}
\]

Thus the speeds are 35 mph and 30 mph.

58. Let \( x \) be amount of the 40% solution and \( y \) be the amount of the 5% solution. To find these values, solve the following system.

\[
\begin{align*}
\begin{array}{c}
\begin{cases}
 x + y = 100 \\
.4x + .05y = 100(.15)
\end{cases}
\end{array}
\end{align*}
\]

Thus the hospital should use approximately 71.4 liters of the 5% solution and 28.6 liters of the 40% solution.

60. Let \( x \) be the number of 2-pointers and \( y \) be the number of 3-pointers. To find these values, solve the following system.

\[
\begin{align*}
\begin{array}{c}
\begin{cases}
 x + y = 17 \\
2x + 3y = 39
\end{cases}
\end{array}
\end{align*}
\]

Thus he scored 12 two pointers and 5 three pointers.
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Cumulative Review

1. \[ n \quad a_n = 2n+1 \]
   \[
   \begin{array}{c|c}
   n & a_n \\
   \hline
   1 & 2(1)+1 = 3 \\
   2 & 2(2)+1 = 5 \\
   3 & 2(3)+1 = 7 \\
   4 & 2(4)+1 = 9 \\
   5 & 2(5)+1 = 11 \\
   \end{array}
   \]

2. \[ a_{50} = 2(50)+1 = 101 \]

3. \[ n \quad a_n = 2^n+1 \]
   \[
   \begin{array}{c|c}
   n & a_n \\
   \hline
   1 & 2^{1+1} = 2^2 = 4 \\
   2 & 2^{2+1} = 2^3 = 8 \\
   3 & 2^{3+1} = 2^4 = 16 \\
   4 & 2^{4+1} = 2^5 = 32 \\
   5 & 2^{5+1} = 2^6 = 64 \\
   \end{array}
   \]

4. \[ (12a+3b)^0 + (12a)^0 + (3b)^0 + 12a^0 + 3b^0 = 1+1+1+12(1)+3(1) = 18 \]

5. \[ (-1)^4 + 4^4 + (1)^4 = 1 + \frac{1}{4} + \frac{1}{4} = 1 + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{9}{4} \]

Section 12.2: Systems of Linear Equations in Three Variables

Quick Review 12.2

1. Check the point \((4, -3)\) in both equations.

   **First Equation**
   \[ 2x + 3y = -1 \]
   \[ 2(4) + 3(-3) = -1 \]
   \[ 8 + (-9) = -1 \]
   \[ -1 = -1 \]

   **Second Equation**
   \[ 6x + 5y = 9 \]
   \[ 6(4) + 5(-3) = 9 \]
   \[ 24 - 15 = 9 \]
   \[ 9 = 9 \]

   The point \((4, -3)\) is a solution to the system of linear equations.

2. **Step 1**
   Solve the first equation for \(y\).
   \[ 2x - y = 5 \]
   \[ \frac{x - y}{2} = 1 \]

   **Step 2**
   Substitute \(2x - 5\) for \(y\) in the second equation and then solve for \(x\).
   \[ \frac{x - \frac{2x - 5}{3}}{2} = 1 \]
   \[ 6\left(\frac{x - \frac{2x - 5}{3}}{2}\right) = 6(1) \]
   \[ 3x - 2(2x - 5) = 6 \]
   \[ 3x - 4x + 10 = 6 \]
   \[ -x = -4 \]
   \[ x = 4 \]

   **Step 3**
   Back-substitute \(4\) for \(x\) in the equation obtained in step 1.
   \[ y = 2x - 5 \]
   \[ y = 2(4) - 5 \]
   \[ y = 3 \]

   Solution: \((4, 3)\)

3. The system represents two parallel lines with different \(y\)-intercepts. It is an inconsistent system.

   The proper choice is **C**.

4. The system represents two lines that are identical. It is a consistent system of dependent equations.

   The proper choice is **B**.

5. The system represents two lines with different slopes. It is a consistent system of independent equations.

   The proper choice is **A**.
Exercises 12.2

2. First, produce a $2 \times 2$ system of equations by eliminating $x$ from equations (1) and (2) and then again from equations (1) and (3).

\[
\begin{align*}
(1) \quad x + 2y + 3z &= 7 \\
(2) \quad x - 3y - 2z &= -13 \\
(3) \quad 2x - y + 2z &= 5 \\
\end{align*}
\]

The $2 \times 2$ system is

\[
\begin{align*}
5y + 5z &= 20 \\
-5y - 4z &= -9
\end{align*}
\]

Use the addition method to solve this $2 \times 2$ system of equations. Add the equations to eliminate $y$.

\[
\begin{align*}
&5y + 5z = 20 \\
&-5y - 4z = -9 \\
\hline
&z = 11
\end{align*}
\]

Back substitute:

\[
\begin{align*}
&5y + 5z = 20 \\
&5y + 5(11) = 20 \\
&5y = -35 \\
&y = -7 \\
&x = -12 \\
&\text{The answer as an ordered triple is } (-12, -7, 11)
\end{align*}
\]

4. First, produce a $2 \times 2$ system of equations by eliminating $x$ from equations (1) and (3) and then again from equations (2) and (3).

\[
\begin{align*}
(1) \quad 3x - y + 2z &= 4 \\
(2) \quad 2x + 2y - z &= 10 \\
(3) \quad x - y + 3z &= -4 \\
\end{align*}
\]

The $2 \times 2$ system is

\[
\begin{align*}
2y - 7z &= 16 \\
4y - 7z &= 18
\end{align*}
\]

Use the addition method to solve this $2 \times 2$ system of equations. Multiply the second equation by $-1$ and add the equations to eliminate $y$.

\[
\begin{align*}
&2y - 7z = 16 \\
&-4y + 7z = -18 \\
\hline
&-2y = -2 \\
&y = 1
\end{align*}
\]

Back substitute:

\[
\begin{align*}
&2(1) - 7z = 16 \\
&-7z = 14 \\
&z = -2 \\
&3x - y + 2z = 4 \\
&3y - (1) + 2(-2) = 4 \\
&x = 3 \\
&\text{The answer as an ordered triple is } (3, 1, -2)
\end{align*}
\]
6. First, produce a $2 \times 2$ system of equations by eliminating $z$ from equations (1) and (2) and then again from equations (2) and (3).

\[
\begin{align*}
(1) & \quad 3x - y + z = 8 \\
(2) & \quad 2x + 3y - z = 0 \\
(3) & \quad 4x + 2y + z = 7
\end{align*}
\]

The $2 \times 2$ system is \[
\begin{align*}
5x + 2y &= 8 \\
6x + 5y &= 7
\end{align*}
\]

Use the addition method to solve this $2 \times 2$ system of equations. Multiply the first equation by 5. Multiply the second equation by $-2$. Add the equations to eliminate $y$:

\[
\begin{align*}
25x + 10y &= 40 \\
-12x - 10y &= -14
\end{align*}
\]

Back substitute:

\[
\begin{align*}
5x + 2y &= 8 \\
2y &= -2 \\
y &= -1
\end{align*}
\]

The answer as an ordered triple is $(2, -1, 1)$.

8. First, produce a $2 \times 2$ system of equations by eliminating $z$ from equations (1) and (3) and then again from equations (2) and (3).

\[
\begin{align*}
(1) & \quad x + 2y + 2z = 4 \\
(2) & \quad 2x + y + 2z = 3 \\
(3) & \quad 3x + y - 4z = 2
\end{align*}
\]

The $2 \times 2$ system is \[
\begin{align*}
x + y &= 2 \\
7x + 3y &= 8
\end{align*}
\]

Use the addition method to solve this $2 \times 2$ system of equations. Multiply the first equation by $-3$. Add the equations to eliminate $y$.

\[
\begin{align*}
-3x - 3y &= -6 \\
7x + 3y &= 8 \\
4x &= 2
\end{align*}
\]

Back substitute:

\[
\begin{align*}
x + y &= 2 \\
\left(\frac{1}{2}\right) + y &= 2 \\
y &= \frac{3}{2}
\end{align*}
\]

The answer as an ordered triple is \[
\left(\frac{1}{2}, \frac{3}{2}, \frac{1}{4}\right)
\]
10. First, produce a $2 \times 2$ system of equations by eliminating $x$ from equations (1) and (3) and combining the resulting equation with (2).

\[
\begin{align*}
(1) & \quad 2x + y = 7 \\
(2) & \quad y - z = 2 \\
(3) & \quad x + z = 2
\end{align*}
\]

The $2 \times 2$ system is

\[
\begin{align*}
(1) & \quad 2x + y = 7 \\
(2) & \quad y - z = 2
\end{align*}
\]

Use the addition method to solve this $2 \times 2$ system of equations. Multiply the first equation by $1$ and add the equations to eliminate $x$.

\[
\begin{align*}
-y + 2z &= -3 \\
y - z &= 2 \\
z &= -1
\end{align*}
\]

Back substitute:

\[
\begin{align*}
(2) & \quad y - z = 2 \\
y - (-1) &= 2 \\
y &= 1
\end{align*}
\]

\[
\begin{align*}
(1) & \quad 2x + y = 7 \\
x &= 3
\end{align*}
\]

The answer as an ordered triple is $(3, 1, -1)$.

12. First, produce a $2 \times 2$ system of equations by eliminating $x$ from equations (1) and (2) and combining the resulting equation with (3).

\[
\begin{align*}
(1) & \quad x + y = 0 \\
(2) & \quad x + 2z = 5 \\
(3) & \quad y + z = 4
\end{align*}
\]

The $2 \times 2$ system is

\[
\begin{align*}
(1) & \quad x + y = 0 \\
(2) & \quad -x - 2z = -5
\end{align*}
\]

Use the addition method to solve this $2 \times 2$ system of equations. Multiply the first equation by $-1$. Add the equations to eliminate $y$.

\[
\begin{align*}
-y + 2z &= 5 \\
y + z &= 4 \\
3z &= 9 \\
z &= 3
\end{align*}
\]

Back substitute:

\[
\begin{align*}
(3) & \quad y + z = 4 \\
y + 3 &= 4 \\
y &= 1
\end{align*}
\]

\[
\begin{align*}
(1) & \quad x + y = 0 \\
x + 1 &= 0 \\
x &= -1
\end{align*}
\]

The answer as an ordered triple is $(-1, 1, 3)$.
14. First, produce a \(2 \times 2\) system of equations by eliminating \(x\) from equations (1) and (2) and then again from equations (1) and (3).

\[
\begin{align*}
(1) & \quad x + 2y + 2z = 2 \\
(2) & \quad 2x - y + z = 1 \\
(3) & \quad 4x + 3y + 5z = 3
\end{align*}
\]

Multiply both sides of the first equation by \(-2\).

The \(2 \times 2\) system is \[
\begin{align*}
-5x - 3z &= -3 \\
-5x - 3z &= -5
\end{align*}
\]

Use the addition method to solve this \(2 \times 2\) system of equations. Multiply the first equation by \(-1\). Add the equations to eliminate \(x\) (and \(z\)).

\[
\begin{align*}
5x + 3z &= 3 \\
-5x - 3z &= -5 \\
0 &= -2
\end{align*}
\]

The resulting equation is a contradiction. Therefore there is no solution. The system is inconsistent.

16. \[
\begin{bmatrix}
1 & -3 & 3 & 10 \\
1 & 1 & -4 & -17 \\
-1 & 1 & 1 & 6
\end{bmatrix}
\]

18. \[
\begin{bmatrix}
6 & -3 & 0 & 2 \\
0 & 3 & 2 & 1 \\
2 & 0 & -1 & 5
\end{bmatrix}
\]

20. \[
\begin{align*}
x + 2y + 3z &= 7 \\
2x - y + 2z &= 5 \\
x - 3y - 2z &= -13
\end{align*}
\]

22. \[
\begin{align*}
x + y &= 0 \\
x + 2z &= 5 \\
y + z &= 4
\end{align*}
\]

24. \[
\begin{align*}
x &= 2 \\
y &= 6 \\
z &= -3
\end{align*}
\]

26. \[
\begin{align*}
\begin{bmatrix}
2 & 3 & 4 & 5 \\
1 & -1 & 3 & 6 \\
3 & -2 & 2 & 10
\end{bmatrix}
\end{align*} \xrightarrow{\eta_i + \eta_2} \begin{align*}
\begin{bmatrix}
1 & -1 & 3 & 6 \\
2 & 3 & 4 & 5 \\
3 & -2 & 2 & 10
\end{bmatrix}
\end{align*}
\]

28. \[
\begin{align*}
\begin{bmatrix}
1 & 2 & 5 & 1 \\
0 & 5 & 6 & 4 \\
0 & 8 & 2 & 5
\end{bmatrix}
\end{align*} \xrightarrow{\eta_1 - \eta_2} \begin{align*}
\begin{bmatrix}
1 & 2 & 5 & 1 \\
0 & 1 & 6 & 4 \\
0 & 8 & 2 & 5
\end{bmatrix}
\end{align*}
\]

30. \[
\begin{align*}
\begin{bmatrix}
1 & 3 & 5 & 11 \\
2 & 7 & 9 & 13 \\
4 & 8 & 3 & 7
\end{bmatrix}
\end{align*} \xrightarrow{\eta_1 + \eta_2 - 4\eta_3} \begin{align*}
\begin{bmatrix}
1 & 3 & 5 & 11 \\
0 & 4 & 0 & -7 \\
0 & -4 & -17 & 37
\end{bmatrix}
\end{align*}
\]

32. \[
\begin{align*}
\begin{bmatrix}
1 & 3 & 5 & 11 \\
0 & 4 & 0 & -7 \\
0 & -4 & -17 & 37
\end{bmatrix}
\end{align*} \xrightarrow{\eta_1 + 3\eta_3} \begin{align*}
\begin{bmatrix}
1 & 3 & 5 & 11 \\
0 & 4 & 0 & -7 \\
0 & 0 & 1 & 1
\end{bmatrix}
\end{align*}
\]

34. \[
\begin{align*}
\begin{bmatrix}
1 & 0 & 2 & -1 \\
0 & 1 & 3 & -8 \\
0 & 0 & 4 & -12
\end{bmatrix}
\end{align*} \xrightarrow{\eta_1 - \eta_2} \begin{align*}
\begin{bmatrix}
1 & 0 & 2 & -1 \\
0 & 1 & 3 & -8 \\
0 & 0 & 1 & -3
\end{bmatrix}
\end{align*}
\]

36. \[
\begin{align*}
\begin{bmatrix}
1 & 0 & 2 & -1 \\
0 & 1 & 3 & -8 \\
0 & 0 & 1 & -3
\end{bmatrix}
\end{align*} \xrightarrow{\eta_1 - 3\eta_3} \begin{align*}
\begin{bmatrix}
1 & 0 & 2 & -1 \\
0 & 1 & 3 & -8 \\
0 & 0 & 1 & -3
\end{bmatrix}
\end{align*}
\]

38. Answer: \((5, -9, 11)\)

40. The last row of the matrix is a contradiction. Thus there is no solution.
42. Since the last row of the matrix gives us an identity, we have a dependent system of equations with an infinite number of solutions. Solving for \( x \) in the first equation we obtain: \( x = 2 + 4z \). Solving for \( y \) in the second equation we obtain: \( y = 3 - 5z \). Thus the general solution of the system is \( (2 + 4z, 3 - 5z, z) \). For \( z = 0, z = 1, \) and \( z = 3 \), we obtain the following particular solutions: \( (2, 3, 0) \), \( (6, -2, 1) \), and \( (14, -12, 3) \).

44.

\[
\begin{bmatrix}
1 & -2 & 5 \\
0 & 3 & -6 \\
0 & 4 & -3
\end{bmatrix}
\xrightarrow{r_1 \leftarrow 5r_3}
\begin{bmatrix}
1 & -2 & 5 \\
0 & 1 & -2 \\
0 & 4 & -3
\end{bmatrix}
\xrightarrow{r_2 \leftarrow r_2 - r_1}
\begin{bmatrix}
1 & 0 & -5 \\
0 & 1 & -2 \\
0 & 4 & -3
\end{bmatrix}
\xrightarrow{r_3 \leftarrow r_3 - 4r_1}
\begin{bmatrix}
1 & 0 & -5 \\
0 & 1 & -2 \\
0 & 0 & 1
\end{bmatrix}
\]

46.

\[
\begin{bmatrix}
1 & 3 & -2 \\
0 & 10 & -5 \\
4 & 2 & -1
\end{bmatrix}
\xrightarrow{r_3 \leftarrow r_3 - 4r_1}
\begin{bmatrix}
1 & 3 & -2 \\
0 & 10 & -5 \\
0 & -10 & 7
\end{bmatrix}
\]

48.

\[
\begin{bmatrix}
1 & 0 & 17 \\
0 & 1 & 5 \\
0 & 2 & -1
\end{bmatrix}
\xrightarrow{r_1 \leftarrow r_1 - 2r_3}
\begin{bmatrix}
1 & 0 & 17 \\
0 & 1 & 5 \\
0 & 0 & 9
\end{bmatrix}
\xrightarrow{r_2 \leftarrow r_2 - 5r_3}
\begin{bmatrix}
1 & 0 & 17 \\
0 & 1 & -12 \\
0 & 0 & 23
\end{bmatrix}
\]

50.

\[
\begin{bmatrix}
1 & 0 & 1 \\
0 & -1 & 1 \\
1 & 0 & 1
\end{bmatrix}
\xrightarrow{r_1 \leftarrow r_1 - r_2}
\begin{bmatrix}
1 & 0 & 0 \\
0 & -1 & 1 \\
1 & 0 & 1
\end{bmatrix}
\xrightarrow{r_2 \leftarrow r_2 + r_1}
\begin{bmatrix}
1 & 0 & 0 \\
0 & 0 & 1 \\
1 & 0 & 1
\end{bmatrix}
\xrightarrow{r_3 \leftarrow r_3 - 2r_1}
\begin{bmatrix}
1 & 0 & 0 \\
0 & 0 & 1 \\
0 & 0 & 1
\end{bmatrix}
\]

52.

Answer: \( (3, 1, -2) \)
### Chapter 12: A preview of College Algebra

**5.4.**

\[
\begin{bmatrix}
1 & -2 & 7 & 3 \\
2 & 2 & -3 & -5 \\
1 & -11 & 24 & 11
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -2 & 7 & 3 \\
0 & 6 & -17 & -11 \\
0 & -9 & 17 & 8
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -2 & 7 & 3 \\
0 & 1 & -17/6 & -11/6 \\
0 & -9 & 17 & 8
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 4/3 & -2/3 \\
0 & 1 & -17/6 & -11/6 \\
0 & 0 & -17/2 & 1
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 0 & -2 \\
0 & 1 & 0 & 1 \\
0 & 0 & 1 & 1
\end{bmatrix}
; \text{ Answer: } (-2, 1, 1)

**5.6.**

\[
\begin{bmatrix}
6 & -3 & 3 & 3 \\
3 & 3 & -1 & 5 \\
5 & 2 & -2 & 4
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -1/2 & 1/2 & 1/2 \\
3 & 3 & -1 & 5 \\
5 & 2 & -2 & 4
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -1/2 & 1/2 & 1/2 \\
0 & 9/2 & -5/2 & 7/2 \\
0 & 9/2 & -5/2 & 7/2
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -1/2 & 1/2 & 1/2 \\
0 & 1 & -5/9 & 7/9 \\
0 & 9/2 & -2 & 2
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 2/3 & 4/3 \\
0 & 1 & 0 & 4/3 \\
0 & 0 & 1 & 1
\end{bmatrix}
; \text{ Answer: }\left(\frac{2}{3}, \frac{4}{3}, 1\right)

**5.8.**

\[
\begin{bmatrix}
1 & -3 & 2 & 1 \\
4 & -2 & 1 & 2 \\
2 & 4 & -2 & 0
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -3 & 2 & 1 \\
0 & 10 & -7 & -2 \\
0 & 10 & -6 & -2
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -3 & 2 & 1 \\
0 & 1 & -7/10 & -1/5 \\
0 & 10 & -6 & -2
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & -10/5 & 2/5 \\
0 & 1 & -7/10 & -1/5 \\
0 & 0 & 1 & 0
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 0 & 2/5 \\
0 & 1 & 0 & 1/5 \\
0 & 0 & 1 & 0
\end{bmatrix}
; \text{ Answer: }\left(\frac{2}{5}, -\frac{1}{5}, 0\right)
Section 12.2: Systems of Linear Equations in Three Variables

60. \[
\begin{bmatrix}
2 & 2 & -2 \\
3 & -1 & 5 \\
4 \\
\end{bmatrix}
\Rightarrow
\begin{bmatrix}
1 & 1 & -1 \\
3 & -1 & 5 \\
0 & -4 & 8 \\
\end{bmatrix}
\Rightarrow
\begin{bmatrix}
1 & 1 & -1 \\
0 & 1 & -2 \\
0 & 1 & -2 \\
\end{bmatrix}
\]
Solving for \(a\) in the first equation we obtain: \(a = 1 - c\). Solving for \(b\) in the second equation we obtain: \(b = -1 + 2c\). Thus the general solution of the system is \((1-c, -1+2c, c)\). For \(c = 0\) and \(c = 1\), we obtain the following particular solutions: \((1, -1, 0)\), and \((0, 1, 1)\).

62. Let \(x\) be the smallest number, \(z\) be the largest number, and \(y\) be the third number.
\[
\begin{cases}
x + y + z = 285 \\
7y + z = 0 \\
7x + y = 0
\end{cases}
\rightarrow
\begin{bmatrix}
1 & 1 & 1 \\
0 & -7 & 1 \\
-7 & 1 & 0 \\
\end{bmatrix}
\Rightarrow
\begin{bmatrix}
1 & 1 & 1 \\
0 & -7 & 1 \\
0 & 8 & 7 \\
\end{bmatrix}
\Rightarrow
\begin{bmatrix}
1 & 1 & 1 \\
0 & 1 & -7 \\
0 & 8 & 7 \\
\end{bmatrix}
\]
The numbers are 5, 35, and 245.

64. Let \(x\) be the smallest side, \(z\) be the largest side, and \(y\) be the third side.
\[
\begin{cases}
x + y + z = 188 \\
x + y - z = 12 \\
x - \frac{1}{2}y = 10
\end{cases}
\rightarrow
\begin{bmatrix}
1 & 1 & 1 \\
1 & 1 & -1 \\
1 & \frac{1}{2} & 0 \\
\end{bmatrix}
\Rightarrow
\begin{bmatrix}
1 & 1 & 1 \\
0 & 0 & -2 \\
0 & 0 & -3 \\
\end{bmatrix}
\Rightarrow
\begin{bmatrix}
1 & 1 & 1 \\
0 & 3 & -2 \\
0 & 0 & -2 \\
\end{bmatrix}
\]
The lengths of the sides are 40 cm, 60 cm, and 88 cm.
Chapter 12: A preview of College Algebra

66. Let $x$ be the first angle, $y$ be the second angle, and $z$ be the third angle.

\[
\begin{align*}
\begin{cases}
x + y + z = 180 \\
x - z = -78 \\
3x - y = 0
\end{cases}
\end{align*}
\]

\[
\begin{array}{c|c|c|c}
 & 1 & 1 & 1 \\
\hline
1 & 1 & 1 & 180 \\
0 & -1 & -2 & -258 \\
0 & -4 & -3 & -540 \\
\end{array}
\]

\[
\begin{array}{c|c|c|c}
 & 1 & 0 & -1 \\
\hline
1 & 0 & -1 & -78 \\
0 & 0 & 1 & 98.4 \\
0 & 0 & 1 & 98.4 \\
\end{array}
\]

The angles are $20.4^\circ$, $61.2^\circ$, and $98.4^\circ$.

68. Let $A$, $B$, and $C$ be the acreage of the three crops.

\[
\begin{align*}
\begin{cases}
120A + 85B + 80C = 26350 \\
4A + 12B + 8C = 2520 \\
500A + 900B + 700C = 210000
\end{cases}
\end{align*}
\]

\[
\begin{align*}
\begin{cases}
\begin{pmatrix}
120 & 85 & 80 \\
4 & 12 & 8 \\
500 & 900 & 700
\end{pmatrix} \rightarrow \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\end{cases}
\end{align*}
\]

The farmer should plant 80 acres of A, 150 acres of B and 50 acres of C.

70. The intercepts of $3x + 2y + 2z = 6$ are $(0, 0, 3)$, $(0, 3, 0)$, and $(2, 0, 0)$. Thus the graph is A.

72. The intercepts of $2x + 3y + 6z = 6$ are $(0, 0, 1)$, $(0, 2, 0)$, and $(3, 0, 0)$.

Group Discussion Questions

73. a. \[
\begin{pmatrix}
3 & -5 & 1 \\
1 & 2 & 4
\end{pmatrix} \rightarrow \begin{pmatrix}
1 & 0 & 2 \\
0 & 1 & 1
\end{pmatrix}; \text{ Answer: (2, 1)}
\]

b. \[
\begin{align*}
\frac{1}{x} &= 2, \quad \frac{1}{y} = 1 \\
\text{or} \quad x &= \frac{1}{2}, \quad y = 1; \text{ Answer: \left(\frac{1}{2}, 1\right)}
\end{align*}
\]

74. a. \[
\begin{pmatrix}
1 & -2 & -1 \\
2 & -1 & 3 \\
-3 & -2 & 2
\end{pmatrix} \rightarrow \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}; \text{ Answer: (1, -3, 2)}
\]

b. \[
\begin{align*}
\frac{1}{x} &= 1, \quad \frac{1}{y} = -3, \quad \frac{1}{z} = 2 \\
\text{or} \quad x &= 1, \quad y = -\frac{1}{3}, \quad z = \frac{1}{2}; \text{ Answer: \left(1, -\frac{1}{3}, \frac{1}{2}\right)}
\end{align*}
\]
Section 12.2: Systems of Linear Equations in Three Variables

75. \[
\begin{align*}
ax + by + cz &= 5 \\
ax - by - cz &= -1 \\
2ax + 3by + 4cz &= 13
\end{align*}
\]
\[
\begin{align*}
a(1)+b(-3)+c(5) &= 5 \\
a(1)-b(-3)-c(5) &= -1 \\
2a(1)+3b(-3)+4c(5) &= 13
\end{align*}
\]
\[
\begin{bmatrix}
1 & -3 & 5 \\
1 & 3 & -5 \\
2 & -9 & 20
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
; \quad \text{Thus } a = 2, \ b = -1, \ c = 0
\]

76. \[
\begin{align*}
\begin{bmatrix}
a & b & c \\
2a & -b & c \\
-a & b & 2c
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & b & c \\
2a & -b & c \\
-a & b & 2c
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & b & c \\
0 & -3b & -c \\
2b & 3c & -21
\end{bmatrix}
\end{align*}
\]
\[
\begin{align*}
\eta' &= \frac{1}{3b^2} \\
\frac{\eta' - \eta}{a} &= \frac{1}{3b^2} \\
\frac{\eta' - \eta}{b} &= \frac{1}{3b^2} \\
\frac{\eta' - \eta}{c} &= \frac{1}{3b^2} \\
\frac{\eta' - \eta}{d} &= \frac{1}{3b^2}
\end{align*}
\]

Cumulative Review

1. \[d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(10 - 2)^2 + (12 - (-3))^2} = \sqrt{8^2 + 15^2} = \sqrt{64 + 225} = \sqrt{289} = 17\]

2. \[c^2 = a^2 + b^2 = (7)^2 + (24)^2 = 49 + 576 = 625\]

3. \[(x-5)(x^2+5x+25) = x^3 + 5x^2 + 25x - 5x^2 - 25x - 125 = x^3 - 125\]

4. The integer factors of 36 between 1 and 36 are: 1, 2, 3, 4, 6, 9, 12, 18, and 36.

5. 37 is prime. Its only factors are 1 and 37.
Chapter 12: A preview of College Algebra

Section 12.3: Horizontal and Vertical Translations of the Graphs of Functions

Quick Review 12.3

Exercises 12.3

2. The graph of \( f(x) = |x + 3| \) is the graph of \( y = |x| \) shifted left 3 units. Thus the answer is B.

4. The graph of \( f(x) = |x - 3| \) is the graph of \( y = |x| \) shifted right 3 units. Thus the answer is A.

6. The graph of \( f(x) = (x - 5)^2 \) is the graph of \( y = x^2 \) shifted right 5 units. Thus the answer is A.

8. The graph of \( f(x) = x^2 + 5 \) is the graph of \( y = x^2 \) shifted up 5 units. Thus the answer is C.

10. The graph of \( f(x) = \sqrt{x + 4} \) is the graph of \( y = \sqrt{x} \) shifted left 4 units. Thus the answer is A.

12. The graph of \( f(x) = \sqrt{x} - 4 \) is the graph of \( y = \sqrt{x} \) shifted down 4 units. Thus the answer is B.

14. The graph of \( f(x) = |x + 1| - 2 \) is the graph of \( y = |x| \) shifted left 1 unit and down up 2 units. Thus the answer is A.

16. The graph of \( f(x) = |x - 1| - 2 \) is the graph of \( y = |x| \) shifted right 1 unit and down 2 units. Thus the answer is C.
18. The graph of $y = f(x) + 2$ is the graph of $y = f(x)$ shifted up two units.

20. The graph of $y = f(x - 1)$ is the graph of $y = f(x)$ shifted right one unit.

22. The graph of $y = f(x) - 3$ is the graph of $y = f(x)$ shifted down three units.

24. The graph shown is the graph of $y = \frac{x}{2}$ shifted down 3 units. Thus the equation is $y = \frac{x}{2} - 3$. The $y$-intercept is $(0, -3)$.

26. The elements in the $y_3$ column are 7 units greater than the elements in the $y_1 = f(x)$ column. Thus the equation for $y_3$ is $y_3 = f(x) + 7$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = f(x) - 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>$0 - 10 = -10$</td>
</tr>
<tr>
<td>-2</td>
<td>$8 - 10 = -2$</td>
</tr>
<tr>
<td>-1</td>
<td>$6 - 10 = -4$</td>
</tr>
<tr>
<td>0</td>
<td>$0 - 10 = -10$</td>
</tr>
<tr>
<td>1</td>
<td>$-4 - 10 = -14$</td>
</tr>
<tr>
<td>2</td>
<td>$0 - 10 = -10$</td>
</tr>
</tbody>
</table>
Chapter 12: A preview of College Algebra

30. \[ \begin{array}{c|c|c|c} x & y_1 = f(x) & y_2 = f(x+1) \\ \hline -3 & -3 & -4 \\ -2 & -2 & -3 \\ -1 & -1 & -2 \\ 0 & 0 & -1 \\ 1 & -1 & 0 \\ 2 & -2 & 1 \\ 3 & -3 & 2 \end{array} \]

For the same \( y \) value in \( y_1 \) and \( y_2 \), the \( x \)-value is 1 unit less for \( y_2 \) than for \( y_1 \).

32. For the same \( y \) value in \( y_1 \) and \( y_3 \), the \( x \)-value is 1 unit less for \( y_3 \) than for \( y_1 \). Thus the equation for \( y_3 \) is \( y_3 = f(x+1) \).

34. The graph of \( f(x) = (x+6)^2 \) is the graph of \( y = x^2 \) shifted left 6 units. Thus the vertex is \((-6, 0)\).

36. The graph of \( f(x) = x^2 - 11 \) is the graph of \( y = x^2 \) shifted down 11 units. Thus the vertex is \((0, -11)\).

38. The graph of \( f(x) = (x-8)^2 + 5 \) is the graph of \( y = x^2 \) shifted right 8 units and up 5 units. Thus the vertex is \((8, 5)\).

40. The graph of \( f(x) = |x| + 14 \) is the graph of \( y = |x| \) shifted up 14 units. Thus the vertex is \((0, 14)\).

42. The graph of \( f(x) = |x-10| \) is the graph of \( y = |x| \) shifted right 10 units. Thus the vertex is \((10, 0)\).

44. The graph of \( f(x) = |x+8| + 9 \) is the graph of \( y = |x| \) shifted left 9 units and up 8 units. Thus the vertex is \((-9, 8)\).

46. A translation of the graph of \( y = f(x) \) eleven units right is obtained in the graph of \( y = f(x-11) \). Answer: C.

48. A translation of the graph of \( y = f(x) \) eleven units up is obtained in the graph of \( y = f(x)+11 \). Answer: F.

50. A translation of the graph of \( y = f(x) \) eleven units down and eleven units left is obtained in the graph of \( y = f(x+11)-11 \). Answer: B.

52. Domain: \([0-5, 5-5] = [-5, 0] \); Range: \([2, 4] \)

54. Domain: \([0, 5] \); Range: \([2+8, 4+8] = [10, 12] \)

56. Domain: \([0+6, 5+6] = [6, 11] \); Range: \([2-4, 4-4] = [-2, 0] \)
Section 12.3: Horizontal and Vertical Translations of the Graphs of Functions

58. The graph of \( f(x) = x + 4 \) is the graph of the linear equation \( y = x \) shifted up 4 units. Answer: C.

60. The graph of \( f(x) = |x| - 3 \) is the graph of the absolute value equation \( y = |x| \) shifted down 3 units. Answer: B.

62. The graph of \( f(x) = (x + 4)^3 \) is the graph of the cubic equation \( y = x^3 \) shifted left 4 units. Answer: G.

64. The graph of \( f(x) = (x - 3)^2 \) is the graph of the quadratic equation \( y = x^2 \) shifted right 3 units. Answer: F.

66. a. 

<table>
<thead>
<tr>
<th>( n )</th>
<th>( a_n = n^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
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</table>

b. 

<table>
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<th>( n )</th>
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<tr>
<td>1</td>
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</tr>
<tr>
<td>2</td>
<td>-5</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>16</td>
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c. 

<table>
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<th>( n )</th>
<th>( a_n = (n-9)^2 )</th>
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<tr>
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</tr>
<tr>
<td>2</td>
<td>49</td>
</tr>
<tr>
<td>3</td>
<td>36</td>
</tr>
<tr>
<td>4</td>
<td>25</td>
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<tr>
<td>5</td>
<td>16</td>
</tr>
</tbody>
</table>

d. 

<table>
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<th>( n )</th>
<th>( a_n = (n-1)^2 + 3 )</th>
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</thead>
<tbody>
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<td>6</td>
</tr>
<tr>
<td>2</td>
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<td>3</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
</tr>
<tr>
<td>5</td>
<td>18</td>
</tr>
</tbody>
</table>

68. 

<table>
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<tr>
<th>( t )</th>
<th>( R(t) = t^2 )</th>
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<tbody>
<tr>
<td>Year</td>
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<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
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<tr>
<td>3</td>
<td>9</td>
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<tr>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
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<table>
<thead>
<tr>
<th>( t )</th>
<th>( R(t) = (t-1995)^2 )</th>
</tr>
</thead>
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</tr>
<tr>
<td>1995</td>
<td>0</td>
</tr>
<tr>
<td>1996</td>
<td>1</td>
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<td>1997</td>
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<td>1998</td>
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<td>1999</td>
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70. 

<table>
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<th>iron</th>
<th>distance for new clubs</th>
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<tbody>
<tr>
<td>2</td>
<td>200 + 5 = 205</td>
</tr>
<tr>
<td>3</td>
<td>190 + 5 = 195</td>
</tr>
<tr>
<td>4</td>
<td>180 + 5 = 185</td>
</tr>
<tr>
<td>5</td>
<td>165 + 5 = 170</td>
</tr>
<tr>
<td>6</td>
<td>150 + 5 = 155</td>
</tr>
</tbody>
</table>

For the same revenue value in the tables, the x-value is 1995 units more for the second table than the first table.

Group Discussion Questions

71. In each case the graph of the second equation is obtained by reflecting the graph of the first equation across the x-axis.

72. The graph of \( f(x) = c|x| \) is obtained by stretching (or compressing if \( 0 < c < 1 \)) the graph of \( f(x) = |x| \) vertically.

73. The graph of \( f(x) = -cx^2 \) is obtained by reflecting the graph of \( f(x) = x^2 \) across the x-axis and stretching (or compressing if \( 0 < c < 1 \)) the graph vertically.

74. Since the graph is that of a parabola, one can conclude that the student typed the function as \( f(x) = (x + 2)^2 + 3 \).
Chapter 12: A preview of College Algebra

Cumulative Review

1. \[ \frac{x}{x-2} + \frac{1}{x} = \frac{x^2 + x - 2}{x(x-2)} = \frac{x^2 + x - 2}{x(x-2)} = \frac{(x+2)(x-1)}{x(x-2)} \]
2. \[ \frac{x}{x-2} + \frac{1}{x-2} = \frac{x^2 + x - 2}{x(x-2)} = \frac{x^2 + x - 2}{x(x-2)} = \frac{1}{x-2} \]

3. \[ \frac{x}{x-2} - \frac{1}{x-1} = \frac{x^2 - 1}{x(x-1)} = \frac{x^2 - 1}{x(x-1)} \]
4. \[ (x - y)^2 - x^2 - y^2 = x^2 - 2xy + y^2 - x^2 - y^2 = -2xy \]

5. \[ 5(2x - 3y) - 4(x - 2y) + 3x - y = 10x - 15y - 4x + 8y + 3x - y = 10x + 3x - 4x - 15y + 8y - y = 9x - 8y \]

Section 12.4: Stretching, Shrinking, and Reflecting Graphs of Functions

Quick Review 12.4

1. \( f(2) = 3(2)^2 - 4 = 3 \cdot 4 - 4 = 12 - 4 = 8 \)
2. \( -f(2) = -(3(2)^2 - 4) = -(3 \cdot 4 - 4) = -(12 - 4) = -8 \)

3. \( f(-2) = 3(-2)^2 - 4 = 3 \cdot 4 - 4 = 12 - 4 = 8 \)
4. \( f(0) = 3(0)^2 - 4 = 3 \cdot 0 - 4 = -4 \)

5. \( -f(0) = -(3(0)^2 - 4) = -(3 \cdot 0 - 4) = -(0) = 4 \)

Exercises 12.4

2. The graph of the absolute value function \( f(x) = -|x| \) is obtained by reflecting the graph of \( f(x) = |x| \) across the \( x \)-axis. Thus the answer is H.

4. The graph of the linear function \( f(x) = -x \) is obtained by reflecting the graph of \( f(x) = x \) across the \( x \)-axis. Thus the answer is A.

6. The graph of the square root function \( f(x) = -\sqrt{x} \) is obtained by reflecting the graph of \( f(x) = \sqrt{x} \) across the \( x \)-axis. Thus the answer is F.

8. The graph of the cubic function \( f(x) = -x^3 \) is obtained by reflecting the graph of \( f(x) = x^3 \) across the \( x \)-axis. Thus the answer is C.
Section 12.4: Stretching, Shrinking, and Reflecting Graphs of Functions

### Table 10.1

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$-f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>5</td>
<td>-5</td>
</tr>
<tr>
<td>-2</td>
<td>0</td>
<td>-0</td>
</tr>
<tr>
<td>-1</td>
<td>-3</td>
<td>-3</td>
</tr>
<tr>
<td>0</td>
<td>-4</td>
<td>-4</td>
</tr>
<tr>
<td>1</td>
<td>-3</td>
<td>-3</td>
</tr>
</tbody>
</table>

14. The graph of the scaled function $f(x) = 5x^2$ can be found by vertically stretching the graph of $y = x^2$ by a factor of 5. Thus the answer is D. (Note that the graph in D is stretched more than the graph in B.)

16. The graph of the scaled function $f(x) = \frac{1}{5}x^2$ can be found by vertically shrinking the graph of $y = x^2$ by a factor of $\frac{1}{5}$. Thus the answer is C. (Note that the graph in C is compressed more than the graph in A.)

18. The graph of the linear function $f(x) = \frac{1}{5}x$ is the graph that has the least positive slope. Thus the answer is B.

20. The graph of the linear function $f(x) = -2x$ is the graph that has the least negative slope. Thus the answer is C.

22. The graph of the scaled function $f(x) = 3|x|$ can be found by vertically stretching the graph of $y = |x|$ by a factor of 3. Thus the answer is B.

24. The graph of the scaled function $f(x) = \frac{1}{4}|x|$ can be found by vertically shrinking the graph of $y = |x|$ by a factor of $\frac{1}{4}$. Thus the answer is C. (Note that the graph in C is compressed more than the graph in D.)

26. The graph of the scaled function $f(x) = \frac{1}{4}\sqrt{x}$ can be found by vertically shrinking the graph of $y = \sqrt{x}$ by a factor of $\frac{1}{4}$. Thus the answer is C.

28. The graph of the scaled function $f(x) = \frac{9}{4}\sqrt{x}$ can be found by vertically stretching the graph of $y = \sqrt{x}$ by a factor of $\frac{9}{4}$. Thus the answer is D. (Note that the graph in B is stretched more than the graph in D.)
30. The graph of \( y = \frac{1}{2} f(x) \) is obtained by vertically shrinking the graph of \( y = f(x) \) by a factor of \( \frac{1}{2} \).

32. The graph of \( y = 3f(x) \) is obtained by vertically stretching the graph of \( y = f(x) \) by a factor of 3, and then reflecting the graph across the x-axis.

34. The graph of \( y = f(x + 2) \) is the graph of \( y = f(x) \) shifted left two units.

36. \( \begin{array}{c|c} x & \frac{1}{3} f(x) \\ \hline -2 & \frac{1}{3} (18) = 6 \\ -1 & \frac{1}{3} (9) = 3 \\ 0 & \frac{1}{3} (3) = 1 \\ 1 & \frac{1}{3} (1) = \frac{1}{3} \\ 2 & \frac{1}{3} (3) = 1 \end{array} \) 

38. \( \begin{array}{c|c} x & -3f(x) \\ \hline -2 & -3(18) = -54 \\ -1 & -3(9) = -27 \\ 0 & -3(3) = -9 \\ 1 & -3(1) = -3 \\ 2 & -3(3) = -9 \end{array} \) 

40. For each value of \( x \), \( y_2 = -y_1 \). Thus \( y_2 = -f(x) \).

42. For each value of \( x \), \( y_2 = 10y_1 \). Thus \( y_2 = 10f(x) \).

44. For each value of \( x \), \( y_2 = y_1 + 5 \). Thus \( y_2 = f(x) + 5 \).

46. For the same \( y \) value in \( y_1 \) and \( y_2 \), the \( x \)-value is 2 units more for \( y_2 \) than for \( y_1 \). Thus the equation for \( y_2 \) is \( y_2 = f(x - 2) \).

48. The graph of \( f(x) = \frac{1}{7} x^2 \) is obtained by vertically shrinking the graph of \( f(x) = x^2 \) by a factor of \( \frac{1}{7} \). The location of the vertex \((0, 0)\) will not change.

50. The graph of \( f(x) = \frac{1}{7} (x - 7)^2 \) is obtained by vertically shrinking the graph of \( f(x) = x^2 \) by a factor of \( \frac{1}{7} \) and then shifting the graph right 7 units. Thus the vertex is \((7, 0)\).
Section 12.4: Stretching, Shrinking, and Reflecting Graphs of Functions

52. The graph of \( f(x) = (x + 6)^2 + 9 \) is obtained by shifting the graph of \( f(x) = x^2 \) left 6 units and up 9 units. Thus the vertex is \((-6, 9)\).

54. The graph of \( f(x) = -(x + 6)^2 - 9 \) is obtained by shifting the reflection of the graph of \( f(x) = x^2 \) left 6 units and down 9 units. Thus the vertex is \((-6, -9)\).

56.

a. The graph of \( f(x) = |x + 2| \) is obtained by shifting \( y = |x| \) left 2 units.

b. The graph of \( f(x) = -|x + 2| \) is obtained by reflecting the graph of \( y = |x + 2| \) across the x-axis.

c. The graph of \( f(x) = -|x + 2| - 3 \) is obtained by shifting the graph of \( y = -|x + 2| \) down three units.

58. A vertical shrinking of \( y = f(x) \) by a factor of \( \frac{1}{7} \) is obtained by multiplying the function by \( \frac{1}{7} \). Answer: C.

60. A horizontal shift of \( y = f(x) \) left 7 units is obtained by adding 7 to \( x \) within the function. Answer: A.

62. A vertical shift of \( y = f(x) \) up 7 units is obtained by adding 7 to the function. Answer: E.

64. Range: \( [2 - 2, 6 - 2] = [0, 4] \)

66. Range: \( \left[\frac{1}{2}(2), \frac{1}{2}(6)\right] = [1, 3) \)

68. Range: \( (-3(6), -3(2)) = (-18, -6] \)
Chapter 12: A preview of College Algebra

70. a. 
\[
\begin{array}{c|c}
 n & a_n = \frac{1}{n} \\
 \hline
 1 & a_1 = \frac{1}{1} = 1 \\
 2 & a_2 = \frac{1}{2} = \frac{1}{2} \\
 3 & a_3 = \frac{1}{3} = \frac{1}{3} \\
 4 & a_4 = \frac{1}{4} = \frac{1}{4} \\
 5 & a_5 = \frac{1}{5} = \frac{1}{5} \\
\end{array}
\]

b. 
\[
\begin{array}{c|c}
 n & a_n = |n - 5| \\
 \hline
 1 & a_1 = |1 - 5| = 4 \\
 2 & a_2 = |2 - 5| = 3 \\
 3 & a_3 = |3 - 5| = 2 \\
 4 & a_4 = |4 - 5| = 1 \\
 5 & a_5 = |5 - 5| = 0 \\
\end{array}
\]

c. 
\[
\begin{array}{c|c}
 n & a_n = -|n - 5| \\
 \hline
 1 & a_1 = -|1 - 5| = -4 \\
 2 & a_2 = -|2 - 5| = -3 \\
 3 & a_3 = -|3 - 5| = -2 \\
 4 & a_4 = -|4 - 5| = -1 \\
 5 & a_5 = -|5 - 5| = 0 \\
\end{array}
\]

d. 
\[
\begin{array}{c|c}
 n & a_n = 2|n - 5| \\
 \hline
 1 & a_1 = 2|1 - 5| = 8 \\
 2 & a_2 = 2|2 - 5| = 6 \\
 3 & a_3 = 2|3 - 5| = 4 \\
 4 & a_4 = 2|4 - 5| = 2 \\
 5 & a_5 = 2|5 - 5| = 0 \\
\end{array}
\]

72. Plane 1

<table>
<thead>
<tr>
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<th>D</th>
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<tbody>
<tr>
<td>hours</td>
<td>distance</td>
</tr>
<tr>
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<td>200</td>
</tr>
<tr>
<td>2</td>
<td>400</td>
</tr>
<tr>
<td>3</td>
<td>600</td>
</tr>
<tr>
<td>4</td>
<td>800</td>
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</table>

Plane 2

<table>
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<th>D</th>
</tr>
</thead>
<tbody>
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<td>hours</td>
<td>distance</td>
</tr>
<tr>
<td>1</td>
<td>2(200) = 400</td>
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<tr>
<td>2</td>
<td>2(400) = 800</td>
</tr>
<tr>
<td>3</td>
<td>2(600) = 1200</td>
</tr>
<tr>
<td>4</td>
<td>2(800) = 1600</td>
</tr>
</tbody>
</table>

74.

Group Discussion Questions

75. a. and b.

c. \( y = (x - 2)^2 - 5 \)

d. \( x = 2 \) when \( y = 0 \)

76. a. and b.

c. \( y = \frac{1}{2} x^2 - 4 \)

d. \( x = -5 \) when \( y = 0 \)

77. The student mistakenly typed \( y = 5|x| \) instead of \( y = 0.5|x| \).

78. The student mistakenly typed \( y = 3\sqrt{x} \) instead of \( y = -3\sqrt{x} \).
Cumulative Review

1. The horizontal line through \((2, -3)\) is \(y = -3\)

2. The vertical line through \((2, -3)\) is \(x = 2\)

3. Use the point-slope form of the line:
   \[ y - y_1 = m(x - x_1) \]
   \[ y - (-3) = \frac{3}{5}(x - 2) \]
   \[ y + 3 = \frac{3}{5}x - \frac{6}{5} \]
   \[ y = \frac{3}{5}x - \frac{21}{5} \]

4. First, find the slope:
   \[ m = \frac{2 - (-3)}{7 - 2} = \frac{5}{5} = 1 \]
   Use the slope intercept form:
   \[ y - y_1 = m(x - x_1) \]
   \[ y - (-3) = 1(x - 2) \]
   \[ y + 3 = x - 2 \]
   \[ y = x - 5 \]

5. Using the point-slope form we know that the equation of the line is:
   \[ y - y_1 = m(x - x_1) \]
   \[ y - (-3) = -\frac{3}{4}(x - 2) \]
   \[ y + 3 = -\frac{3}{4}x + \frac{6}{4} \]
   \[ y + 3 = -\frac{3}{4}x + \frac{6}{4} \]
   \[ y = -\frac{3}{4}x + \frac{3}{2} \]
   \[ y = -\frac{3}{4}x + \frac{3}{2} \]
   With \(x = 6\) we have
   \[ y = -\frac{3}{4}(6) - \frac{3}{2} = -\frac{9}{2} - \frac{3}{2} = -\frac{12}{2} = -6 \]

Section 12.5: Algebra of Functions

Quick Review 12.5

1. \((2x^2 - 7x - 15) + (2x + 3) = 2x^2 - 7x - 15 + 2x + 3\)
   \[= 2x^2 - 7x + 2x - 15 + 3\]
   \[= 2x^2 - 5x - 12\]

2. \((2x^2 - 7x - 15) - (2x + 3) = 2x^2 - 7x - 15 - 2x - 3\)
   \[= 2x^2 - 7x - 2x - 15 - 3\]
   \[= 2x^2 - 9x - 18\]

3. \((2x^2 - 7x - 15)(2x + 3)\)
   \[= 4x^3 + 6x^2 - 14x^2 - 21x - 30x - 45\]
   \[= 4x^3 - 8x^2 - 51x - 45\]

4. \(\frac{2x^2 - 7x - 15}{2x + 3} = \frac{(2x + 3)(x - 5)}{2x + 3} = x - 5\)
5. First, write \( y = 2x + 3 \).
   Next, swap all values of \( x \) and \( y \).
   \( x = 2y + 3 \)

   Now, solve for \( y \).
   \( x = 2y + 3 \)
   \( x - 3 = 2y \)
   \( y = \frac{x - 3}{2} \)

   The inverse of \( f(x) \) is \( f^{-1}(x) = \frac{x - 3}{2} \).

Exercises 12.5

2. a. \((f + g)(-3) = f(-3) + g(-3)\)
   \(=((-3)^2 - 1) + (2(-3) + 5)\)
   \(=(8) + (-1) = 7\)

   b. \((f - g)(-3) = f(-3) - g(-3)\)
   \(=((-3)^2 - 1) - (2(-3) + 5)\)
   \(=(8) - (-1) = 9\)

   c. \((f \cdot g)(-3) = f(-3) \cdot g(-3)\)
   \(=((-3)^2 - 1) \cdot (2(-3) + 5)\)
   \(=(8) \cdot (-1) = -8\)

   d. \(\left(\frac{f}{g}\right)(-3) = \frac{f(-3)}{g(-3)}\)
   \(=((-3)^2 - 1)\)
   \(=\frac{2(-3) + 5}{2(-3) + 5}\)
   \(=\frac{8}{1} = -8\)

4. \(\begin{array}{c|c}
   x & f - g \\
   \hline
   0 & (-2) - (-1) = 1 \\
   3 & (0) - (5) = -5 \\
   8 & (7) - (9) = -2 \\
   \end{array}\)

6. \(\begin{array}{c|c}
   x & f \cdot g \\
   \hline
   0 & (-2) \cdot (-1) = 2 \\
   3 & (0) \cdot (5) = 0 \\
   8 & (7) \cdot (9) = 63 \\
   \end{array}\)

8. \(\begin{array}{c|c}
   x & \frac{f}{g} \\
   \hline
   0 & \frac{(-2)}{(-1)} = 2 \\
   3 & \frac{(0)}{(5)} = 0 \\
   8 & \frac{(7)}{(9)} = \frac{7}{9} \\
   \end{array}\)

10. \(\begin{array}{c|c}
   x & f + g \\
   \hline
   -2 & 3 + 4 = 7 \\
   1 & 5 + (-1) = 4 \\
   4 & 7 + 6 = 13 \\
   \end{array}\)
    \(f + g = \{(-2,7), (1,4), (4,13)\}\)
Section 12.5: Algebra of Functions

12. \( f \cdot g \) 
   \[
   \begin{array}{c|c}
   x & f \cdot g \\
   \hline
   -2 & 3 \cdot 4 = 12 \\
   1 & 5 \cdot (-1) = -5 \\
   4 & 7 \cdot 6 = 42 \\
   \end{array}
   \]
   \( f \cdot g = \{(-2, 12), (1, -5), (4, 42)\} \)

14. \( f - g \) 
   \[
   \begin{array}{c|c}
   x & f - g \\
   \hline
   -3 & 1 - 2 = -1 \\
   -1 & 2 - (-1) = 3 \\
   1 & 3 - 1 = 2 \\
   3 & (-2) - 1 = -3 \\
   5 & 1 - 1 = 0 \\
   \end{array}
   \]

16. \( x \) 
   \[
   \begin{array}{c|c}
   x & \frac{f}{g} \\
   \hline
   -3 & \frac{1}{2} \\
   -1 & \frac{2}{-1} = -2 \\
   1 & \frac{3}{1} = 3 \\
   3 & \frac{-2}{1} = -2 \\
   5 & \frac{1}{1} = 1 \\
   \end{array}
   \]

18. \( g(x) = 1 - \frac{1}{x} = \frac{x - 1}{x} \). The functions are equal. (They have the same domains.)

20. The functions are not equal. (They have different domains.)

22. \( g(x) = \frac{5x^2 + 15}{x^2 + 3} = \frac{5(x^2 + 3)}{x^2 + 3} = 5 \). The functions are equal. (They have the same domains.)

24. a. \( (f \circ g)(-3) = f(g(-3)) \)
    \[= f(2(-3) + 5)\]
    \[= f(-1) = (-1)^2 - 1 = 0\]

b. \( (g \circ f)(-3) = g(f(-3)) \)
    \[= g((-3)^2 - 1)\]
    \[= g(8) = 2(8) + 5 = 21\]

c. \( (f \circ f)(-3) = f(f(-3)) \)
    \[= f((-3)^2 - 1)\]
    \[= f(8) = (8)^2 - 1 = 63\]

d. \( (g \circ g)(-3) = g(g(-3)) \)
    \[= g(2(-3) + 5)\]
    \[= g(-1) = 2(-1) + 5 = 3\]

26. a. \( x \longrightarrow g(x) \longrightarrow f(g(x)) \)
    \[
    \begin{array}{c|c|c}
    x & g(x) & f(g(x)) \\
    \hline
    3 & 6 & 2 \\
    5 & 1 & 3 \\
    2 & 4 & 5 \\
    \end{array}
    \]
    \( f \circ g)(x) = \{(3, 2), (5, 3), (2, 5)\} \)
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b.  \[ x \rightarrow f(x) \rightarrow g(f(x)) \quad x \rightarrow g(f(x)) \quad (f \circ g)(x) = \{(1,6),(4,1),(6,4)\} \]

\[
\begin{align*}
1 & \rightarrow 3 \rightarrow 6 & 1 & \rightarrow 6 \\
4 & \rightarrow 5 \rightarrow 1 & 4 & \rightarrow 1 \\
6 & \rightarrow 2 \rightarrow 4 & 6 & \rightarrow 4 \\
\end{align*}
\]

28.  \( (f \circ g)(x) = f(g(x)) = f(4x + 3) = 3(4x + 3) - 4 = 12x + 9 - 4 = 12x + 5 \)

\( (g \circ f)(x) = g(f(x)) = g(3x - 4) = 4(3x - 4) + 3 = 12x - 16 + 3 = 12x - 13 \)

30.  \( (f \circ g)(x) = f(g(x)) = f(x - 8) = |x - 8| \)

\( (g \circ f)(x) = g(f(x)) = g(|x|) = |x| - 8 \)

32.  \( (f \circ g)(x) = f(g(x)) = f(x^2 + 1) = \frac{1}{x^2 + 1} \)

\( (g \circ f)(x) = g(f(x)) = g\left(\frac{1}{x}\right) + 1 = \frac{1}{x} + 1 \)

34.  \( (f + g)(x) = (2x^2 - x - 3) + (2x - 3) = 2x^2 - x - 3 + 2x - 3 = 2x^2 + x - 6; \quad \text{Domain: } (-\infty, +\infty) \text{ or } \mathbb{R} \)

\( (f - g)(x) = (2x^2 - x - 3) - (2x - 3) = 2x^2 - x - 3 - 2x + 3 = 2x^2 - 3x; \quad \text{Domain: } (-\infty, +\infty) \text{ or } \mathbb{R} \)

\( (f \cdot g)(x) = (2x^2 - x - 3) \cdot (2x - 3) = 2x^2 (2x - 3) - x(2x - 3) - 3(2x - 3) \\
= 4x^3 - 6x^2 - 2x^2 + 3x - 6x + 9 = 4x^3 - 8x^2 - 3x + 9; \quad \text{Domain: } (-\infty, +\infty) \text{ or } \mathbb{R} \)

\( \left(\frac{f}{g}\right)(x) = \frac{2x^2 - x - 3}{2x - 3} = \frac{2x - 3}{2x - 3} \left(\frac{x + 1}{x(1)}\right) = x + 1; \quad \text{Domain: } \left(-\infty, \frac{3}{2}\right] \cup \left(\frac{3}{2}, +\infty\right) \text{ or } \mathbb{R} \sim \left[\frac{3}{2}\right] \)

\( (f \circ g)(x) = f(g(x)) = f(2x - 3) = 2(2x - 3)^2 - (2x - 3) - 3 = 2(4x^2 - 12x + 9) - 2x + 3 - 3 \\
= 2(4x^2 - 12x + 9) - 2x + 3 - 3 = 8x^2 - 24x + 18 - 2x = 8x^2 - 26x + 18; \quad \text{Domain: } (-\infty, +\infty) \text{ or } \mathbb{R} \)

36.  \( (f + g)(x) = \frac{x}{x + 1} + \frac{1}{x} = \frac{x^2 + (x + 1)}{x(x + 1)} = \frac{x^2 + x + 1}{x(x + 1)}; \quad \text{Domain: } (-\infty, -1) \cup (-1, 0) \cup (0, +\infty) \text{ or } \mathbb{R} - \{-1, 0\} \)

\( (f - g)(x) = \frac{x}{x + 1} - \frac{1}{x} = \frac{x^2 - (x + 1)}{x(x + 1)} = \frac{x^2 - x - 1}{x(x + 1)}; \quad \text{Domain: } (-\infty, -1) \cup (-1, 0) \cup (0, +\infty) \text{ or } \mathbb{R} - \{-1, 0\} \)

\( (f \cdot g)(x) = \frac{x}{x + 1} \cdot \frac{1}{x} = \frac{x}{x(x + 1)} = \frac{1}{x + 1}; \quad \text{Domain: } (-\infty, -1) \cup (-1, 0) \cup (0, +\infty) \text{ or } \mathbb{R} - \{-1, 0\} \)

\( \left(\frac{f}{g}\right)(x) = \frac{x}{x + 1} \cdot \frac{1}{x} = \frac{x}{x + 1} = \frac{x^2}{x + 1}; \quad \text{Domain: } (-\infty, -1) \cup (-1, 0) \cup (0, +\infty) \text{ or } \mathbb{R} - \{-1, 0\} \)
Section 12.5: Algebra of Functions

\[(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{x}\right) = \frac{1}{x-1} + \frac{x}{(1/x)+1} = \frac{x}{1+x};\]

Domain: \((-\infty, -1) \cup (-1, 0) \cup (0, +\infty)\) or \(\mathbb{R} \setminus \{-1, 0\}\)

38.

40. \((f \circ f^{-1})(x) = f(f^{-1}(x)) = f\left(\frac{x+3}{2}\right) = \left(x + \frac{x}{2}\right) - 3 = \frac{x}{2} - 3 = x\)

\((f^{-1} \circ f)(x) = f^{-1}(f(x)) = f^{-1}(2x) - 3 = \frac{2x-3}{2} + 3 = \frac{2x}{2} = x\)

42. a. First, replace \(f(x)\) with \(y\).

\[
\begin{align*}
f(x) &= 4x + 5 \\
y &= 4x + 5
\end{align*}
\]

Next, exchange \(x\) and \(y\), then solve for \(y\).

\[
\begin{align*}
x &= 4y + 5 \\
y &= \frac{x-5}{4}
\end{align*}
\]

Rewrite the inverse using functional notation.

\[
f^{-1}(x) = \frac{x-5}{4}
\]

44. \(h(x) = (f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = \left(\sqrt{x}\right)^2 + 1 = x + 1, \quad x \geq 0\)

48. Let \(g(x) = 3x^2 - 7x + 9\) (the “inside” function) and \(f(x) = \frac{1}{x}\) (the “outside” function)

52. a. \(F(500) = 5,000\)

b. \(C(500) = 5,000 + 252,500 = 257,500\)

c. \(C(u) = F(u) + V(u) = 5000 + u^2 + 5u = u^2 + 5u + 5000\)

d. \(A(500) = \frac{C(500)}{500} = \frac{257,500}{500} = 515\)

e. \(A(u) = \frac{C(u)}{u} = \frac{u^2 + 5u + 5000}{u}\)

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54. a. \( N(7) = 36(7) - 7^2 = 203 \) items 
   b. \( P(7) = 5(7) + 45 = 80 \) 
   c. \( R(7) = N(7) \cdot P(7) = 203 \cdot 80 = 16,240 \) 
   d. \( R(m) = N(m) \cdot P(m) = (36m - m^2)(5m + 45) = -5m^3 + 135m^2 + 1620m \)

56. a. \( C(5000) = 0.30(5000) + 400 = 1900 \); The cost of making 5000 doses is $1,900. 
   b. \( R(1900) = 1.5(1900) = 2850 \); The revenue derived from selling $1900 worth of the vaccine is $2,850. 
   c. \( (R \circ C)(5000) = R(C(5000)) = R(1900) = 2,850 \); The revenue derived from selling 5000 doses of the vaccine is $2,850.
   d. \( (R \circ C)(d) = R(C(d)) = R(0.30d + 400) = 1.5(0.30d + 400) = 0.45d + 600 \)

58. a. \( A(w) = 576 - w^2 \) 
   b. \( w(x) = 24 - 2x \) 
   c. \( (A \circ w)(x) = A(w(x)) = A(24 - 2x) = 576 - (24 - 2x)^2 = 576 - (576 - 96x + 4x^2) = 576 - 576 + 96x - 4x^2 = 96x - 4x^2 \) 
   \( (A \circ w)(x) \) is the area of the board when \( x \) cm is trimmed from all sides.

Group Discussion Questions

60. \( (f \circ g)(x) = f(g(x)) = \log(10^x) = x \); The functions \( f(x) \) and \( g(x) \) are inverses of each other

61. a. \( f^{-1} = \{(4, 1), (5, 2), (3, 3), (1, 4), (2, 5)\} \). Yes, \( f(x) = f^{-1}(x) \) for all input values \( x \).
   
   b. \( f = \{(1, 7), (2, 6), (3, 5), (4, 4), (5, 3), (6, 2), (7, 1)\} \)
   
   c. The inverse of \( f(x) = \frac{1}{x} \) is \( f^{-1}(x) = \frac{1}{x} \); Yes.
   
   d. Answers will vary.

62. a. \( f(x) = \frac{3x - 2}{7x - 3} \) 
   \[ y = \frac{3x - 2}{7x - 3} \] 
   Exchange \( x \) and \( y \) then solve for \( y \).
   \[ x = \frac{3y - 2}{7y - 3} \] 
   \[ x(7y - 3) = 3y - 2 \] 
   \[ 7xy - 3x = 3y - 2 \] 
   \[ 7xy - 3y = 3x - 2 \] 
   \[ y(7x - 3) = 3x - 2 \] 
   \[ y = \frac{3x - 2}{7x - 3} \] Thus \( f^{-1}(x) = \frac{3x - 2}{7x - 3} \) 

b. The functions \( f(x) \) and \( f^{-1}(x) \) are identical.

c. The graph of \( y = f(x) \) is symmetric about the line \( y = x \) .
Cumulative Review

1. The additive identity is 0.
2. The multiplicative identity is 1.
3. The additive inverse of 6 is \(-6\).
4. The multiplicative inverse of 6 is \(\frac{1}{6}\).
5. The property that justifies rewriting \(7x(3x-2)+5(3x-2)\) as \((7x+5)(3x-2)\) is the distributive property of multiplication over addition.

Quick Review 12.6

1. \(a_n = 3n + 5\)
2. \(a_n = 4n + 8\)
3. \(n \quad a_n = n^2 - 5n + 8\)
4. \(n \quad a_n = 5 \cdot 2^{n-1}\)

Exercises 12.6

2. a. The sequence is arithmetic.
   The common difference is \(d = (124 - 144) = (104 - 124) = ... = (44 - 64) = -20\).
   \[d = (124 - 144) = (104 - 124) = ... = (44 - 64) = -20.\]
   b. The sequence is geometric. The common ratio is \(r = \frac{72}{144} = \frac{36}{72} = ... = \frac{4.5}{9} = \frac{1}{2}\).
   c. The sequence is geometric. The common ratio is \(r = \frac{-9}{9} = \frac{9}{-9} = ... = \frac{-9}{9} = -1\).
   d. The sequence is neither arithmetic nor geometric. (The sequence does not have a common ratio or common difference.)

4. a. The sequence: 1, 5, 25, 125, 625,... is geometric. The common ratio is \(r = \frac{5}{1} = \frac{25}{5} = ... = \frac{625}{125} = ... = 5\).
   b. The sequence: 2, 4, 16, 256, 65536,... is neither arithmetic nor geometric. (The sequence does not have a common ratio or common difference.)
   c. The sequence: 3, 8, 13, 18, 23, 28,... is arithmetic.
   The common difference is \(d = (8 - 3) = (13 - 8) = ... = (28 - 23) = ... = 5\).
   d. The sequence: 4, 4, 4, 4, 4,... is arithmetic with a common difference of \(d = (4 - 4) = (4 - 4) = ... = (4 - 4) = ... = 0\).
   It is also geometric with a common ratio of \(r = \frac{4}{4} = \frac{4}{4} = ... = \frac{4}{4} = ... = 1\).
6. a. \( a_1 = 4 \)
\( a_2 = a_1 + 5 = (4) + 5 = 9 \)
\( a_3 = a_2 + 5 = (9) + 5 = 14 \)
\( a_4 = a_3 + 5 = (14) + 5 = 19 \)
\( a_5 = a_4 + 5 = (19) + 5 = 24 \)
\( a_6 = a_5 + 5 = (24) + 5 = 29 \)
Answer: 4, 9, 14, 19, 24, 29,...

b. \( a_1 = 4 \)
\( a_2 = 5(a_1) = 5(4) = 20 \)
\( a_3 = 5(a_2) = 5(20) = 100 \)
\( a_4 = 5(a_3) = 5(100) = 500 \)
\( a_5 = 5(a_4) = 5(500) = 2500 \)
\( a_6 = 5(a_5) = 5(2500) = 12500 \)
Answer: 4, 20, 100, 500, 2500, 12500,...

c. \( a_1 = 4 \)
\( a_2 = -(a_1) = -(4) = -4 \)
\( a_3 = -(a_2) = -(-4) = 4 \)
\( a_4 = -(a_3) = -(4) = -4 \)
\( a_5 = -(a_4) = -(-4) = 4 \)
\( a_6 = -(a_5) = -(4) = -4 \)
Answer: 4, 4, 4, 4, 4, 4,...

d. \( a_1 = 1 \)
\( a_2 = 2 \)
\( a_3 = 2a_1 + a_2 = 2(1) + (2) = 4 \)
\( a_4 = 2a_2 + a_3 = 2(2) + (4) = 8 \)
\( a_5 = 2a_3 + a_4 = 2(4) + (8) = 16 \)
\( a_6 = 2a_4 + a_5 = 2(8) + (16) = 32 \)
Answer: 1, 2, 4, 8, 16, 32,...

8. a. \( a_1 = -8 \)
\( a_2 = -8 + 4 = -4 \)
\( a_3 = -4 + 4 = 0 \)
\( a_4 = 0 + 4 = 4 \)
\( a_5 = 4 + 4 = 8 \)
\( a_6 = 8 + 4 = 12 \)
Answer: -8, -4, 0, 4, 8, 12,...

c. \( a_1 = 11 - 2(1) = 9 \)
\( a_2 = 11 - 2(2) = 7 \)
\( a_3 = 11 - 2(3) = 5 \)
\( a_4 = 11 - 2(4) = 3 \)
\( a_5 = 11 - 2(5) = 1 \)
\( a_6 = 11 - 2(6) = -1 \)
Answer: 9, 7, 5, 3, 1, -1,...

b. \( a_1 = 4(1) - 1 = 3 \)
\( a_2 = 4(2) - 1 = 7 \)
\( a_3 = 4(3) - 1 = 11 \)
\( a_4 = 4(4) - 1 = 15 \)
\( a_5 = 4(5) - 1 = 19 \)
\( a_6 = 4(6) - 1 = 23 \)
Answer: 3, 7, 11, 15, 19, 23,...

d. \( a_1 = 1.2 \)
\( a_2 = 1.2 + 0.4 = 1.6 \)
\( a_3 = 1.6 + 0.4 = 2 \)
\( a_4 = 2 + 0.4 = 2.4 \)
\( a_5 = 2.4 + 0.4 = 2.8 \)
\( a_6 = 2.8 + 0.4 = 3.2 \)
Answer: 1.2, 1.6, 2.0, 2.4, 2.8, 3.2,...
10. a. \( a_1 = 3 \)
\[ a_2 = 2 \cdot 3 = 6 \\
 a_3 = 2 \cdot 6 = 12 \\
 a_4 = 2 \cdot 12 = 24 \\
 a_5 = 2 \cdot 24 = 48 \\
\]
Answer: 3, 6, 12, 24, 48, ...

b. \( a_1 = 9 \left( \frac{1}{3} \right)^1 = 3 \)
\[ a_2 = 9 \left( \frac{1}{3} \right)^2 = 1 \\
 a_3 = 9 \left( \frac{1}{3} \right)^3 = \frac{1}{3} \\
 a_4 = 9 \left( \frac{1}{3} \right)^4 = \frac{1}{9} \\
 a_5 = 9 \left( \frac{1}{3} \right)^5 = \frac{1}{27} \\
\]
Answer: 3, \( \frac{1}{3} \), \( \frac{1}{9} \), \( \frac{1}{27} \), ...

c. \( a_1 = 96 \left( -\frac{1}{2} \right)^1 = -48 \\
 a_2 = 96 \left( -\frac{1}{2} \right)^2 = 24 \\
 a_3 = 96 \left( -\frac{1}{2} \right)^3 = -12 \\
 a_4 = 96 \left( -\frac{1}{2} \right)^4 = 6 \\
 a_5 = 96 \left( -\frac{1}{2} \right)^5 = -3 \\
\]
Answer: -48, 24, -12, 6, -3, ...

d. \( a_1 = 10 \)
\[ a_2 = 2a_1 = 2(10) = 20 \\
 a_3 = 2a_2 = 2(20) = 40 \\
 a_4 = 2a_3 = 2(40) = 80 \\
 a_5 = 2a_4 = 2(80) = 160 \\
\]
Answer: 10, 20, 40, 80, 160, ...

12. The \( n^{\text{th}} \) term for a geometric sequence is determined by \( a_n = a_1 r^{n-1} \).

a. \( a_n = a_1 r^{n-1} \)
\[ a_{10} = 4 \left( \frac{1}{256} \right) \left( 2 \right)^{10-1} = 2 \\
\]
c. \( r = \frac{40}{8} = 5 \\
 a_{32} = 5a_{31} \\
 a_{32} = 5(40) = 200 \\
\]

14. \( a_n = a_1 + (n-1)d \)
\[ 496 = 48 + (n-1)(7) \\
 448 = (n-1)(7) \\
 n-1 = 64 \\
 n = 65 \\
\]
18. \( a_{35} = a_1 + (15)d \)
   \[28 = 23 + 15d\]
   \[15d = 5\]
   \[d = \frac{1}{3}\]

20. \( a_n = a_1 + (n-1)d \)
   \[6 = -12 + (n-1)\left(\frac{1}{2}\right)\]
   \[18 = (n-1)\left(\frac{1}{2}\right)\]
   \[n-1 = 36\]
   \[n = 37\]

22. \( a_n = a_1 r^{n-1} \)
   \[1 = 1024 \left(\frac{1}{2}\right)^{n-1}\]
   \[
   \frac{1}{1024} = \left(\frac{1}{2}\right)^{n-1}
   \]
   \[
   \left(\frac{1}{2}\right)^{10} = \left(\frac{1}{2}\right)^{n-1}
   \]
   \[n-1 = 10\]
   \[n = 11\]

24. \( a_n = a_1 r^{n-1} \)
   \[64 = a_1 (4)^{n-1}\]
   \[64 = a_1 (1024)\]
   \[a_1 = \frac{1}{16}\]

26. \( a_{47} = a_{45} r^2 \)
   \[425 = 17 (r)^2\]
   \[r^2 = 25\]
   \[r = 5 \text{ or } -5\]

28. \( a_n = a_1 r^{n-1} \)
   \[a_8 = 7000 (0.1)^{8-1} = 0.0007\]

30. a. \[
   \sum_{k=4}^{8} \frac{k+3}{5} = \frac{1+3}{5} + \frac{2+3}{5} + \frac{3+3}{5} + \frac{4+3}{5} + \frac{5+3}{5} + \frac{6+3}{5} + \frac{7+3}{5} + \frac{8+3}{5} = 6
   \]
   b. \[
   \sum_{j=2}^{6} (j^2 + 1) = (4^2 + 1) + (5^2 + 1) + (6^2 + 1) + (7^2 + 1) + (8^2 + 1) = 17 + 26 + 37 + 50 + 65 = 195
   \]
   c. \[
   \sum_{j=4}^{7} 3^j = 3^4 + 3^5 + 3^6 + 3^7 = 81 + 243 + 729 + 2187 = 3240
   \]
   d. \[
   \sum_{k=10}^{16} 7 = 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 = 49
   \]

32. \( S_n = \frac{n}{2} (a_1 + a_n) \)
   \[
   S_{40} = \frac{51}{2} (3 + 153) = 3978
   \]

34. \( S_n = \frac{n}{2} (a_1 + a_n) \)
   \[
   S_{40} = \frac{18}{2} (.36 + .64) = 9
   \]

36. \( S_n = \frac{n}{2} [2a_1 + (n-1)d] \)
   \[
   S_{11} = \frac{11}{2} [2(11) + (11-1)(-3)] = -44
   \]

38. \( \sum_{k=1}^{13} (3k - 2) = 1 + 4 + 7 + 10 + ... + 139 \)
   \[
   S_n = \frac{n}{2} (a_1 + a_n)
   \]
   \[
   S_{11} = \frac{47}{2} (1 + 139) = 3290
   \]
40. \[ \sum_{j=1}^{40} \frac{j-5}{3} = \left( -\frac{4}{3} \right) + \left( -\frac{3}{3} \right) + \left( -\frac{2}{3} \right) + \ldots + \left( \frac{35}{3} \right) \]
\[ S_n = \frac{n(n+1)}{2}(a_1 + a_n) \]
\[ S_n = \frac{40(40+1)}{2} \left( \frac{4}{3} + \frac{35}{3} \right) = \frac{620}{3} \]

44. \[ S_n = \frac{a_1(1-r^n)}{1-r} \]
\[ S_n = 0.5 \left( 1 - 0.1^n \right) \]
\[ S_n = \frac{0.5 \left( 1 - 0.1^n \right)}{1 - 0.1} = 0.555555 \]

48. \[ S_n = \frac{a_1 - ra_n}{1-r} \]
\[ S_n = \frac{64 \left( \frac{1}{2} \right) \left( \frac{1}{8} \right)}{1 - \left( \frac{1}{2} \right)} = \frac{1023}{8} \]

52. \[ S_n = \frac{a_1(1-r^n)}{1-r} \]
\[ S_n = 81 \left( \frac{1}{3} \right)^n \]
\[ S_n = \frac{81 \left( \frac{1}{3} \right)^n}{1 - \left( \frac{1}{3} \right)} = \frac{364}{3} \]

56. Since \(|r| < 1\) we may use the following formula to evaluate the infinite geometric series.
\[ S = \frac{a_1}{1-r} \]
\[ S = \frac{7}{1 - \left( \frac{2}{5} \right)} = 5 \]

58. Since \(|r| < 1\) we may use the following formula to evaluate the infinite geometric series.
\[ S = \frac{a_1}{1-r} \]
\[ S = \frac{0.9}{1 - \left( \frac{1}{10} \right)} = 1 \]

60. Since \(|r| < 1\) we may use the following formula to evaluate the infinite geometric series.
\[ S = \frac{a_1}{1-r} \]
\[ S = \frac{\left( \frac{3}{7} \right)}{1 - \left( \frac{3}{7} \right)} = \frac{3}{4} \]

62. Since \(|r| < 1\) we may use the following formula to evaluate the infinite geometric series.
\[ S = \frac{a_1}{1-r} \]
\[ S = \frac{16}{1 - \left( \frac{3}{4} \right)} = \frac{64}{7} \]
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64. Since \( |r| < 1 \) we may use the following formula to evaluate the infinite geometric series.

\[
S = \frac{a_1}{1 - r}
\]

\[
S = \frac{-36}{1 - \left( -\frac{5}{9} \right)} = \frac{162}{7}
\]

66. 

\( a. \) \( 0.555... = 0.5 + 0.05 + 0.005 + ... \)

\[
S = \frac{a_1}{1 - r} = \frac{0.5}{1 - 0.1} = \frac{5}{9}
\]

\( b. \) \( 0.363636... = 0.36 + 0.0036 + 0.000036 + ... \)

\[
S = \frac{a_1}{1 - r} = \frac{0.36}{1 - 0.01} = \frac{4}{11}
\]

\( c. \) \( 0.495495495... = 0.495 + 0.00495 + 0.0000495 + ... \)

\[
S = \frac{a_1}{1 - r} = \frac{0.495}{1 - 0.001} = \frac{55}{111}
\]

\( d. \) \( 8.333... = 8 + 0.3 + 0.03 + 0.003 + ... \)

\[
S = 8 + \frac{a_1}{1 - r} = 8 + \frac{3}{1 - 0.1} = \frac{25}{3}
\]

68. \( S_n = \frac{n}{2} (a_1 + a_n) \)

\[
240 = \frac{n}{2} (4 + 16)
\]

\[
240 = 10n
\]

\[
n = 24
\]

70. \( S_n = \frac{n}{2} (a_1 + a_n) \)

\[
527 = \frac{n}{2} (15 + a_n)
\]

\[
n = 62
\]

\[
a_n = 47
\]

72. \( S_n = \frac{n}{2} (2a_1 + (n-1)d) \)

\[
60 = \frac{25}{2} (2(17) + (25-1)d)
\]

\[
\frac{24}{5} = 34 + 24d
\]

\[
24d = -\frac{146}{5}
\]

\[
d = -\frac{73}{60}
\]
### Section 12.6: Sequences, Series, and Summation Notation

74. \[ S_n = \frac{a_1(1-r^n)}{1-r} \]

46872 = \( \frac{12(1-5^n)}{1-5} \)

\( 1-5^3 = -15624 \)

\( 5^3 = 15625 \)

\( \log 5^3 = \log 15625 \)

\( n \log 5 = \log 15625 \)

\( n = \frac{\log 15625}{\log 5} = 6 \)

76. \[ S = \frac{a_1}{1-r} \]

\[ 27 = \frac{12}{1-r} \]

\[ 27(1-r) = 12 \]

\[ 27 - 27r = 12 \]

\[ -27r = -15 \]

\[ r = \frac{5}{9} \]

80. Find the number of rolls in the stack is:

\[ 40 + 32 + 24 + 16 + 8 = 120 \]

Thus there are 5 layers in the stack.

82. Given the following arithmetic series, find \( S_{20} \):

\[ 100 + 98 + 96 + 94 + \ldots \]

\[ S_n = \frac{n}{2} (2a_1 + (n-1)d) \]

\[ S_{20} = \frac{20}{2} (2(100) + (20-1)(-2)) = 1620 \text{ seats} \]

84. For the total distance traveled down find \( S_n \) in the following geometric series.

\[ 36 + 36(0.6) + 36(0.6)^2 + 36(0.6)^3 + \ldots \]

\[ S_n = \frac{a_1(1-r^n)}{1-r} \]

\[ S_n = \frac{36(1 - (0.6)^3)}{1-0.6} \approx 88.49 \text{ meters} \]

For the total distance traveled up find \( S_n \) in the following geometric series.

\[ 36(0.6) + 36(0.6)^2 + 36(0.6)^3 + 36(0.6)^4 + \ldots \]

\[ S_n = \frac{a_1(1-r^n)}{1-r} \]

\[ S_n = \frac{36(0.6)(1 - (0.6)^3)}{1-0.6} \approx 53.09 \text{ meters} \]

Thus the total distance traveled is approximately:

\[ 88.49 + 53.09 = 141.58 \text{ meters} \]
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86. Determine the following infinite sum.
\[ 3 + \frac{2}{3} + \frac{2}{3}^2 + \frac{2}{3}^3 + \frac{2}{3}^4 + \ldots \]
\[ S = \frac{\frac{2}{3}}{1 - r} \]
\[ S = \frac{3}{1 - \frac{2}{3}} = 9 \text{ meters} \]

Group Discussion Questions

88. If the people stood in a row and the person on the end walked through the line shaking everyone’s hand, then a total of 19 handshakes would have taken place. If the next person were to do the same then there would be an additional 18 handshakes to count. If this were to continue then the total of \(19 + 18 + 17 + 16 + \ldots + 3 + 2 + 1\) handshakes.

Thus the total number of handshakes is \(\frac{19}{2}(1+19) = 190\).

89. a. \(0.333... = \frac{1}{3}\)
   b. \(0.666... = \frac{2}{3}\)
   c. \(0.888... = \frac{8}{9}\)
   d. The three dots imply the digits repeat without an end.
   e. \(0.999... = 1\)
   f. There aren’t any real numbers between 0.999... and 1.
   g. By multiplying both sides by 3 we obtain \(1 = 0.999...\)
   h. The statement is true.

90. The total amount saved is given in the geometric series.
\[ 0.01 + 0.01(2) + 0.01(2)^2 + 0.01(2)^3 + \ldots + 0.01(2)^{n-1} = \frac{0.01(1-2^n)}{1-2} \]
To find the number of days it would take to become a millionaire we solve the following equation.
\[ \frac{0.01(1-2^n)}{1-2} = 1,000,000 \]
\[-0.01(1-2^n) = 1,000,000 \]
\[1-2^n = -100,000,000 \]
\[2^n = 100,000,001 \]
\[n \ln(2) = \ln(100,000,001) \]
\[n = \frac{\ln(100,000,001)}{\ln(2)} \approx 26.6 \text{ days} \]

Thus, you would have to save for a total of 27 days to be a millionaire.
On the 27 day you would be saving \(0.01(2)^{27-1} = 5671,088.64\)
Cumulative Review

1. \(12,000 = 1.2 \times 10^4\)

2. \(0.0045 = 4.5 \times 10^{-3}\)

3. \(-3 \leq x < 5\)

4. \(x \leq 2\)

5. \([2, 7]\)

Section 12.7: Conic Sections

Quick Review 12.7

1. The vertex of the parabola defined by
   \[y = ax^2 + bx + c\] has an \(x\)-coordinate of \(x = -\frac{b}{2a}\)
   and a \(y\)-coordinate of \(f\left(-\frac{b}{2a}\right)\).

2. The \(x\)-coordinate of the vertex of
   \[f(x) = 2x^2 + 4x + 3\] is
   
   \[x = -\frac{b}{2a} = -\frac{4}{2(2)} = -1.\]
   
   The \(y\)-coordinate is
   
   \[f(-1) = 2(-1)^2 + 4(-1) + 3 = 2 - 4 + 3 = 1\]
   
   Vertex: \((-1, 1)\)

3. \(9(x - 2)^2 + 4(y + 1)^2 - 36\)
   
   \[= 9(x^2 - 4x + 4) + 4(y^2 + 2y + 1) - 36\]
   
   \[= 9x^2 - 36x + 36 + 4y^2 + 8y + 4 - 36\]
   
   \[= 9x^2 + 4y^2 - 36x + 8y + 36 + 4 - 36\]
   
   \[= 9x^2 + 4y^2 - 36x + 8y + 4\]

4. \(2x^2 + 5x + 7 = 0\)
   
   \[\frac{2x^2}{2} + \frac{5x}{2} = \frac{-7}{2}\]
   
   \[x^2 + \frac{5}{2}x + \frac{25}{16} = \frac{-7}{2} + \frac{25}{16}\]
   
   \[\left(x + \frac{5}{4}\right)^2 = \frac{-31}{16}\]
   
   \[x + \frac{5}{4} = \pm \sqrt{\frac{-31}{16}}\]
   
   \[x = -\frac{5}{4} \pm i\sqrt{31}\]
   
   \[x = -\frac{5 \pm i\sqrt{31}}{4}\]

5. \(f(x) = x^2 + 12x - 30\)
   
   \[= x^2 + 12x + 36 - 30 - 36\]
   
   \[= (x^2 + 12x + 36) - 30 - 36\]
   
   \[= (x + 6)^2 - 66\]
Exercises 12.7

2. Distance: 
\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]
\[ = \sqrt{(6 - (-6))^2 + (3 - 2)^2} \]
\[ = \sqrt{12^2 + (1)^2} \]
\[ = \sqrt{144 + 1} = \sqrt{145} \approx 12.04 \]

Midpoint: 
\[ \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{-6 + 6}{2}, \frac{2 + 3}{2} \right) \]
\[ = \left( 0, \frac{5}{2} \right) = \left( 0, \frac{5}{2} \right) \]

4. Distance: 
\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]
\[ = \sqrt{((a+1)-(a-3))^2 + (b-(b+3))^2} \]
\[ = \sqrt{(a+1-a+3)^2 + (b-b-3)^2} \]
\[ = \sqrt{(4)^2 + (-3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5 \]

Midpoint: 
\[ \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \]
\[ = \left( \frac{a+1+(a-3)}{2}, \frac{b+(b+3)}{2} \right) \]
\[ = \left( \frac{2a-2}{2}, \frac{2b+3}{2} \right) \]
\[ = \left( \frac{2}{2}, \frac{2b+3}{2} \right) = \left( 1, \frac{2b+3}{2} \right) \]

6. Use the distance formula:
\[ \text{diameter} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]
\[ = \sqrt{(15 - (-8))^2 + (0 - 0)^2} \]
\[ = \sqrt{23^2 + 0^2} = \sqrt{23^2} = 23 \]

8. The equation \( y = x^2 + 4 \) is in the standard form of a parabola. Thus the graph is E.

10. The equation \( \frac{x^2}{4} + \frac{y^2}{1} = 1 \) is in the standard form of an ellipse. Thus the graph is C.

12. The equation \( y = x + 4 \) is in the slope intercept form of a line. Thus the graph is A.

14. Substitute: \( (h, k) = (5, -4) \) and \( r = 4 \) in the standard form of the equation of a circle.
\[ (x - h)^2 + (y - k)^2 = r^2 \]
\[ (x - 5)^2 + (y - (-4))^2 = 4^2 \]
\[ (x - 5)^2 + (y + 4)^2 = 16 \]

18. Write the equation in the standard form of the equation of a circle.
\[ x^2 + y^2 = 144 \]
\[ (x - 0)^2 + (y - 0)^2 = 12^2 \]
The circle has a center \((0, 0)\) and radius 12.

16. Substitute: \( (h, k) = (1, 1) \) and \( r = 0.1 \) in the standard form of the equation of a circle.
\[ (x - h)^2 + (y - k)^2 = r^2 \]
\[ (x - 1)^2 + (y - 1)^2 = 0.1^2 \]
\[ (x - 1)^2 + (y - 1)^2 = 0.01 \]

20. Write the equation in the standard form of the equation of a circle.
\[ (x - 4)^2 + (y + 5)^2 = 36 \]
\[ (x - 4)^2 + (y - (-5))^2 = 6^2 \]
The circle has a center \((4, -5)\) and radius 6.
22. Write the equation in the standard form of the equation of a circle by completing the square.
\[ x^2 + y^2 + 12y - 13 = 0 \]
\[ x^2 + y^2 + 12y + 36 = 13 + 36 \]
\[ x^2 + (y + 6)^2 = 49 \]
\[ (x - 0)^2 + (y - (-6))^2 = 7^2 \]
The circle has a center \((0, -6)\) and radius 7.

24. Write the equation in the standard form of the equation of a circle by completing the square.
\[ 4x^2 + 4y^2 - 8x - 40y + 103 = 0 \]
\[ 4x^2 - 8x + 4y^2 - 40y = -103 \]
\[ 4(x^2 - 2x) + 4(y^2 - 10y) = -103 \]
\[ 4(x^2 - 2x + 1) + 4(y^2 - 10y + 25) = -103 + 4(1) + 4(25) \]
\[ 4(x - 1)^2 + 4(y - 5)^2 = 1 \]
\[ (x - 1)^2 + (y - 5)^2 = \frac{1}{4} \]
\[ (x - 1)^2 + (y - 5)^2 = \left(\frac{1}{2}\right)^2 \]
The circle has a center \((1, 5)\) and radius \(\frac{1}{2}\).

26. Write the equation in the standard form of the equation of an ellipse.
\[ \frac{(x - 1)^2}{9} + \frac{(y + 2)^2}{49} = 1 \]
\[ \frac{(x - 1)^2}{9} + \frac{(y - (-2))^2}{49} = 1 \]
Center: \((1, -2)\)
\[ a^2 = 49 \quad b^2 = 9 \]
\[ a = 7 \quad b = 3 \]
Length of major axis: \(2a = 2(7) = 14\)
Length of major axis: \(2b = 2(3) = 6\)

28. Write the equation in the standard form of the equation of an ellipse.
\[ 36x^2 + 49y^2 = 1764 \]
\[ \frac{(x - 0)^2}{49} + \frac{(y - 0)^2}{36} = 1 \]
Center: \((0, 0)\)
\[ a^2 = 49 \quad b^2 = 36 \]
\[ a = 7 \quad b = 6 \]
Length of major axis: \(2a = 2(7) = 14\)
Length of major axis: \(2b = 2(6) = 12\)
Chapter 12: A preview of College Algebra

30. \((h, k) = (0, 0)\)
\[a = 5; \quad b = 3\]
\[
\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1
\]
\[
\frac{(x-0)^2}{5^2} + \frac{(y-0)^2}{3^2} = 1
\]
\[
x^2 + y^2 = 1
\]

32. \((h, k) = (-5, 2)\)
\[a = \left(\frac{1}{2}\right); \quad b = 6; \quad c = \left(\frac{1}{2}\right)\]
\[
\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1
\]
\[
\frac{(x+5)^2}{9} + \frac{(y-2)^2}{4} = 1
\]

34. \((h, k) = (-3, -4)\)
\[a = \left(\frac{1}{2}\right); \quad b = 8\]
\[
\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1
\]
\[
\frac{(x+3)^2}{4^2} + \frac{(y+4)^2}{8^2} = 1
\]
\[
\frac{1}{16} + \frac{1}{16} = 1
\]

36. \((h, k) = (2, 5)\)
\[a = 4; \quad b = 2\]
\[
\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1
\]
\[
\frac{(x-2)^2}{4^2} + \frac{(y-5)^2}{16} = 1
\]

38. Write the equation in the standard form of the equation of a hyperbola.
\[
\frac{(y - 0)^2}{36} - \frac{(x - 0)^2}{16} = 1
\]
Center: \((0, 0)\)
\[a^2 = 36; \quad b^2 = 16\]
\[a = 6; \quad b = 4\]
The hyperbola opens vertically.

40. Write the equation in the standard form of the equation of a hyperbola.
\[
\frac{(x - 4)^2}{49} - \frac{(y - 6)^2}{9} = 1
\]
Center: \((4, 6)\)
\[a^2 = 49; \quad b^2 = 9\]
\[a = 7; \quad b = 3\]
The hyperbola opens horizontally.
42. Write the equation in the standard form of the equation of a hyperbola.

\[ 16x^2 + 96x = 9y^2 - 126y + 153 \]
\[ 16x^2 + 96x - 9y^2 + 126y = 153 \]
\[ 16(x^2 + 6x) - 9(y^2 - 14y) = 153 \]
\[ 16(x + 3)^2 - 9(y - 7)^2 = -144 \]
\[ \frac{(y - 7)^2}{16} - \frac{(x + 3)^2}{9} = 1 \]

Center: \((-3, 7)\)

\[ a^2 = 16 \quad b^2 = 9 \]
\[ a = 4 \quad b = 3 \]

The hyperbola opens vertically.

44. \((h, k) = (0, 0)\)

\[ a = \left(\frac{1}{2}\right) 12 = 6; \quad b = \left(\frac{1}{2}\right) 6 = 3 \]
\[ \frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1 \]
\[ \frac{(x - 0)^2}{6^2} - \frac{(y - 0)^2}{3^2} = 1 \]
\[ \frac{x^2}{36} - \frac{y^2}{9} = 1 \]

46. \((h, k) = (0, 0)\)

\[ a = 5; \quad b = \left(\frac{1}{2}\right) 16 = 8 \]
\[ \frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1 \]
\[ \frac{(y - 0)^2}{5^2} - \frac{(x - 0)^2}{8^2} = 1 \]
\[ \frac{y^2}{25} - \frac{x^2}{64} = 1 \]
52. a. The solutions to the system of equations are the points of intersection: \((-2, 1)\) and \((6, -7)\).

54. a. The solution to the system of equations is the point of intersection: \((0, 1)\).
57.

58. a. \([-11.75, 11.75, 1] \) by \([-7.75, 7.75, 1]\)

b. \(\frac{x^2}{25} + \frac{y^2}{16} = 1\)

\(\frac{y^2}{16} = 1 - \frac{x^2}{25}\)

\(y = \pm \sqrt{1 - \frac{x^2}{25}}\)

\(y = \pm 4 \sqrt{1 - \frac{x^2}{25}}\)

\([-11.75, 11.75, 1] \) by \([-7.75, 7.75, 1]\)
Cumulative Review

1. \[3(2x-1)+5 = 7(4x-5)-34\]
   \[6x-3+5 = 28x-35-34\]
   \[6x+2 = 28x-69\]
   \[-22x = -71\]
   \[x = \frac{71}{22}\]

2. \[|2x-1| = 25\]
   is equivalent to
   \[2x-1 = -25\] or \[2x-1 = 25\]
   \[2x = -24\] \[2x = 26\]
   \[x = -12\] \[x = 13\]

3. \[(2x-1)^2 = 25\]
   \[2x-1 = \pm\sqrt{25}\]
   \[2x-1 = \pm5\]
   \[2x = 1 \pm 5\]
   \[x = \frac{1 \pm 5}{2}\]
   \[x = -2\] or \[x = 3\]

4. \[\frac{3x+2}{2x-7} = 4\]
   \[(2x-7)\left(\frac{3x+2}{2x-7}\right) = (2x-7)4\]
   \[3x+2 = 8x-28\]
   \[-5x = -30\]
   \[x = 6\]

5. \[\log(3x+1) = 2\]
   \[\log_{10}(3x+1) = 2\]
   is equivalent to \[3x+1 = 10^2\]
   \[3x+1 = 100\]
   \[3x = 99\]
   \[x = 33\]

Review Exercises for Chapter 12

1. a. \[
\begin{bmatrix}
2 & -5 & 17 \\
3 & 4 & 14
\end{bmatrix}
\]
   or \[
\begin{bmatrix}
3 & -4 & 2 \\
2 & 3 & -3
\end{bmatrix}
\]
   b. \[
\begin{bmatrix}
2 & -5 & 17 \\
3 & 4 & 14
\end{bmatrix}
\]

2. a. Answer: \((-2, 6)\)
   b. Answer: \((4, 5, 8)\)

3. Solving for \(x\) in the first equation we obtain: \(x = 5-3z\) . Solving for \(y\) in the second equation we obtain: \(y = 4+2z\) . Thus the general solution of the system is \((5-3z, 4+2z, z)\) . For \(z = 0, z = 1, \) and \(z = 5,\) we obtain the following particular solutions: \((5, 4, 0), (2, 6, 1), \) and \((-10, 14, 5)\).

4. \[
\begin{bmatrix}
3 & 5 & 13 \\
1 & 4 & 2
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 4 & 2 \\
3 & 5 & 13
\end{bmatrix}
\]

5. \[
\begin{bmatrix}
2 & 4 & -10 \\
3 & 2 & -3
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 2 & -5 \\
3 & 2 & -3
\end{bmatrix}
\]

6. \[
\begin{bmatrix}
1 & 2 & 14 \\
3 & -1 & 13
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 2 & 14 \\
0 & -7 & -29
\end{bmatrix}
\]

7. \[
\begin{bmatrix}
1 & 1 & 3 & -6 \\
0 & 1 & 2 & 3
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & -8 & -15 \\
0 & 1 & 2 & 3
\end{bmatrix}
\]
   or \[
\begin{bmatrix}
1 & 1 & 3 & -6 \\
0 & 1 & 2 & 3
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & -8 & -15 \\
0 & 1 & 2 & 3
\end{bmatrix}
\]
Review Exercises for Chapter 12

8. \[
\begin{bmatrix}
1 & 2 & 17 \\
2 & 5 & 41
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 2 & 17 \\
0 & 1 & 7
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 3 \\
0 & 1 & 7
\end{bmatrix} ; \text{ Answer: (3, 7)}
\]

9. \[
\begin{bmatrix}
3 & -2 & -16 \\
1 & 3 & 13
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 3 & 13 \\
3 & -2 & -16
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 3 & 13 \\
0 & -11 & -55
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 3 & 13 \\
0 & 1 & 5
\end{bmatrix} ; \text{ Answer: (-2, 5)}
\]

10. \[
\begin{bmatrix}
3 & 4 & -1 \\
2 & -3 & 5
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 4 & -1 \\
2 & -3 & 5
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -3 & -1 \\
0 & -17 & -3
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -3 & -1 \\
0 & 1 & -1
\end{bmatrix} ; \text{ Answer: (1, -1)}
\]

11. \[
\begin{bmatrix}
1 & -5 & 8 \\
-3 & 15 & 10
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -5 & 8 \\
0 & 0 & 34
\end{bmatrix} ; \text{ The last row is a contradiction. Thus there is no solution.}
\]

12. \[
\begin{bmatrix}
1 & 2 & -1 \\
2 & -1 & 1 \\
3 & 2 & 2
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 2 & -1 \\
0 & -5 & 3 \\
0 & -4 & 5
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 2 & -1 \\
0 & 1 & -3 \\
0 & -4 & 5
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 1 \\
0 & 1 & -3 \\
0 & 0 & 1
\end{bmatrix} ; \text{ Answer: (1, -1, 1)}
\]

13. \[
\begin{bmatrix}
1 & 0 & -3 \\
2 & 1 & 0 \\
0 & 2 & 1
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & -3 \\
0 & 1 & 6 \\
0 & 2 & 1
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & -3 \\
0 & 1 & 6 \\
0 & 0 & 1
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & -3 \\
0 & 1 & 6 \\
0 & 0 & 1
\end{bmatrix} ; \text{ Answer: (1, 4, -3)}
\]

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14. \[
\begin{bmatrix}
2 & -3 & 4 & 11 \\
3 & 2 & -4 & -4 \\
4 & 1 & -3 & -1
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -\frac{3}{2} & 2 & \frac{11}{2} \\
3 & 2 & -4 & -4 \\
4 & 1 & -3 & -1
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -\frac{3}{2} & 2 & \frac{11}{2} \\
3 & 2 & -4 & -4 \\
4 & 1 & -3 & -1
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & -\frac{4}{13} & \frac{10}{13} \\
0 & 1 & \frac{20}{13} & -\frac{41}{13} \\
0 & 0 & \frac{3}{13} & -\frac{12}{13}
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 0 & 2 \\
0 & 1 & 0 & 3 \\
0 & 0 & 1 & 4
\end{bmatrix}; \text{ Answer: } (2, 3, 4)
\]

15. The intercepts of \(5x + 2y + 2.5z = 10\) are \((0, 0, 4), (0, 5, 0), \) and \((2, 0, 0)\).

16. Let \(x\) be the weight of a pallet of bricks boat and \(y\) be the weight of a pallet of concrete blocks. To find these values, solve the following system.
\[
\begin{align*}
2x + 3y &= 18000 \\
4x + y &= 19600
\end{align*}
\rightarrow
\begin{bmatrix}
2 & 3 & 18000 \\
4 & 1 & 19600
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 4080 \\
0 & 1 & 3280
\end{bmatrix}
\]
Thus a pallet of bricks weighs 4,080 pounds and a pallet of concrete blocks weighs 3,280 pounds.

17. \[
\begin{align*}
A + 2B + 5C &= 156 \\
2A + 4B + 8C &= 294 \\
3A + 3B + 7C &= 321
\end{align*}
\rightarrow
\begin{bmatrix}
1 & 2 & 5 & 156 \\
2 & 4 & 8 & 294 \\
3 & 3 & 7 & 321
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 0 & 61 \\
0 & 1 & 0 & 25 \\
0 & 0 & 1 & 9
\end{bmatrix}
\]
The prices are $61.00 for calculator A, $25.00 for calculator B, and $9 for calculator C.
18. a. The graph of \( f(x) = x^2 \) has its vertex at the origin. Thus the graph is \( B \).

b. The graph of \( f(x) = x^2 - 5 \) is the graph of \( y = x^2 \) shifted down 5 units. Thus the graph is \( A \).

c. The graph of \( f(x) = x^2 + 5 \) is the graph of \( y = x^2 \) shifted up 5 units. Thus the graph is \( C \).

19. a. The graph of \( f(x) = \sqrt{x} \) passes through the origin. Thus the graph is \( A \).

b. The graph of \( f(x) = \sqrt{x - 4} \) is the graph of \( f(x) = \sqrt{x} \) shifted right 4 units. Thus the graph is \( C \).

c. The graph of \( f(x) = \sqrt{x + 4} \) is the graph of \( f(x) = \sqrt{x} \) shifted left 4 units. Thus the graph is \( B \).

20. a. The graph of \( f(x) = |x| + 2 \) is the graph of \( y = |x| \) shifted right 1 unit and up two units. Thus the graph is \( B \).

b. The graph of \( f(x) = |x - 2| + 1 \) is the graph of \( y = |x| \) shifted right 2 units and up 1 unit. Thus the graph is \( A \).

c. The graph of \( f(x) = |x + 2| - 1 \) is the graph of \( y = |x| \) shifted left 2 units and down one unit. Thus the graph is \( C \).

21. a. The graph of \( y = f(x - 3) \) is the graph of \( y = f(x) \) shifted right 3 units.

b. The graph of \( y = f(x) + 4 \) is the graph of \( y = f(x) \) shifted up 4 units.

c. The graph of \( y = f(x - 3) + 4 \) is the graph of \( y = f(x) \) shifted right 3 units and up 4 units.
22. a. The graph of \( f(x) = x^2 + 7 \) is obtained by shifting the graph of \( f(x) = x^2 \) up 7 units. Thus the vertex is \((0, 7)\).

b. The graph of \( f(x) = (x - 9)^2 \) is obtained by shifting the graph of \( f(x) = x^2 \) right 9 units. Thus the vertex is \((9, 0)\).

c. The graph of \( f(x) = (x + 11)^2 - 12 \) is obtained by shifting the graph of \( f(x) = x^2 \) left 11 units and down 12 units. Thus the vertex is \((-11, -12)\).

23. a. \[
\begin{array}{c|c|c|c}
 x & y = f(x) + 5 & y = f(x + 5) \\
-2 & 5 + 5 = 10 & -7 & 5 \\
-1 & 9 + 5 = 14 & -6 & 9 \\
0 & 3 + 5 = 8 & -5 & 3 \\
1 & -4 + 5 = 1 & -4 & -4 \\
2 & 0 + 5 = 5 & -3 & 0 \\
\end{array}
\]

b. \[
\begin{array}{c|c|c|c}
 x & y = f(x) + 5 & y = f(x + 5) \\
-2 & 5 + 5 = 10 & -7 & 5 \\
-1 & 9 + 5 = 14 & -6 & 9 \\
0 & 3 + 5 = 8 & -5 & 3 \\
1 & -4 + 5 = 1 & -4 & -4 \\
2 & 0 + 5 = 5 & -3 & 0 \\
\end{array}
\]

24. a. The graph of \( y = f(x) - 17 \) is obtained by shifting the graph of \( y = f(x) \) down 17 units. Answer: D.

b. The graph of \( y = f(x - 1) \) is obtained by shifting the graph of \( y = f(x) \) right 1 unit. Answer: A.

c. The graph of \( y = f(x) + 17 \) is obtained by shifting the graph of \( y = f(x) \) up 17 units. Answer: B.

d. The graph of \( y = f(x + 17) \) is obtained by shifting the graph of \( y = f(x) \) left 17 units. Answer: C.

25. a. To find the table for \( y = f(x) + 2 \) we add 2 units to every \( y \) value in the table for \( f(x) \). Answer: D.

b. To find the table for \( y = f(x + 2) \) we subtract 2 units from every \( x \) value in the table for \( f(x) \). Answer: B.

c. To find the table for \( y = f(x - 1) + 3 \) we add 1 unit to every \( x \) value and add 3 units to every \( y \) value in the table for \( f(x) \). Answer: C.

d. To find the table for \( y = f(x + 3) - 1 \) we subtract 3 units from every \( x \) value and subtract 1 unit from every \( y \) value in the table for \( f(x) \). Answer: A.

26. a. The graph of \( y = -f(x) \) is obtained by reflecting the graph of \( y = f(x) \) across the x-axis. Answer: B.

b. The graph of \( y = 2f(x) \) is obtained by vertically stretching the graph of \( y = f(x) \) by a factor of 2. Answer: C.

c. The graph of \( y = -2f(x) \) is obtained by vertically stretching the graph of \( y = f(x) \) by a factor of 2 and reflecting the graph across the x-axis. Answer: D.

d. The graph of \( y = \frac{1}{2}f(x) \) is obtained by vertically shrinking the graph of \( y = f(x) \) by a factor of \( \frac{1}{2} \). Answer: A.

27. a. The graph of \( y = -f(x) \) is obtained by reflecting the graph of \( y = f(x) \) across the x-axis. Answer: C.

b. The graph of \( y = -\frac{1}{3}f(x) \) is obtained by vertically shrinking the graph of \( y = f(x) \) by a factor of \( \frac{1}{3} \) and reflecting the graph across the x-axis. Answer: D.

c. The graph of \( y = \frac{1}{3}f(x) \) is obtained by vertically shrinking the graph of \( y = f(x) \) by a factor of \( \frac{1}{3} \). Answer: A.
Review Exercises for Chapter 12

28. a. \( y = -f(x) \)
   
<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = -f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(-f(0) = 0)</td>
</tr>
<tr>
<td>1</td>
<td>(-f(8) = -8)</td>
</tr>
<tr>
<td>2</td>
<td>(-f(4) = -4)</td>
</tr>
<tr>
<td>3</td>
<td>(-f(12) = -12)</td>
</tr>
<tr>
<td>4</td>
<td>(-f(24) = -24)</td>
</tr>
</tbody>
</table>

b. \( y = \frac{1}{4} f(x) \)
   
<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = \frac{1}{4} f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(\frac{1}{4}(0) = 0)</td>
</tr>
<tr>
<td>1</td>
<td>(\frac{1}{4}(4) = 2)</td>
</tr>
<tr>
<td>2</td>
<td>(\frac{1}{4}(8) = 2)</td>
</tr>
<tr>
<td>3</td>
<td>(\frac{1}{4}(12) = 3)</td>
</tr>
<tr>
<td>4</td>
<td>(\frac{1}{4}(24) = 6)</td>
</tr>
</tbody>
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c. \( y = 4f(x) \)
   
<table>
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<th>( y = 4f(x) )</th>
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<tr>
<td>0</td>
<td>(4(0) = 0)</td>
</tr>
<tr>
<td>1</td>
<td>(4(8) = 32)</td>
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<tr>
<td>2</td>
<td>(4(4) = 16)</td>
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<td>(4(12) = 48)</td>
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<td>4</td>
<td>(4(24) = 96)</td>
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d. \( y = -3f(x) \)
   
<table>
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<tr>
<th>( x )</th>
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<tbody>
<tr>
<td>0</td>
<td>(-3(0) = 0)</td>
</tr>
<tr>
<td>1</td>
<td>(-3(8) = -24)</td>
</tr>
<tr>
<td>2</td>
<td>(-3(4) = -12)</td>
</tr>
<tr>
<td>3</td>
<td>(-3(12) = -36)</td>
</tr>
<tr>
<td>4</td>
<td>(-3(24) = -72)</td>
</tr>
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29. a. The graph of \( y = 2f(x) \) is obtained by vertically stretching the graph of \( y = f(x) \) by a factor of 2.
   
   Answer: B.

b. The graph of \( y = -f(x) \) is obtained by reflecting the graph of \( y = f(x) \) across the \( x \)-axis.
   
   Answer: A.

c. The graph of \( y = f(x + 2) \) is obtained by shifting the graph of \( y = f(x) \) left 2 units.
   
   Answer: B.

d. The graph of \( y = f(x) + 2 \) is obtained by shifting the graph of \( y = f(x) \) up 2 units.
   
   Answer: C.

30. a. \( f(10) = (3(10) - 7) = 23 \)
   
   b. \( g(10) = 2(10)^2 + 1 = 201 \)
   
   c. \( (f + g)(10) = f(10) + g(10) = 23 + 201 = 224 \)
   
   d. \( (f - g)(10) = f(10) - g(10) = 23 - 201 = -178 \)

31. a. \( (f \cdot g)(2) = f(2) \cdot g(2) \)
   
   \[ = (3(2) - 7) \cdot \left(2(2)^2 + 1\right) \]
   
   \[ = (-1) \cdot (9) = -9 \]

b. \( \left(\frac{f}{g}\right)(2) = \frac{f(2)}{g(2)} = \frac{3(2) - 7}{2(2)^2 + 1} \)
   
   \[ = \frac{-1}{9} = \frac{1}{9} \]

c. \( \left(\frac{g}{f}\right)(2) = \frac{g(2)}{f(2)} = \frac{2(2)^2 + 1}{3(2) - 7} \)
   
   \[ = \frac{9}{-1} = -9 \]

d. \( (g - f)(2) = g(2) - f(2) \)
   
   \[ = (2(2)^2 + 1) - (3(2) - 7) \]
   
   \[ = (9) - (9) = 10 \]
Chapter 12: A preview of College Algebra

32. a. \((f \circ g)(2) = f(g(2))\)
   \[= f(2^2 + 1)\]
   \[= f(9) = 3(9) - 7 = 20\]

   b. \((g \circ f)(2) = g(f(2))\)
   \[= g(3(2) - 7)\]
   \[= g(-1) = 2(-1)^2 + 1 = 3\]

   c. \((f \circ f)(2) = f(f(2))\)
   \[= f(3(2) - 7)\]
   \[= f(-1) = 3(-1) - 7 = -10\]

   d. \((g \circ g)(2) = g(g(2))\)
   \[= g(2^2 + 1)\]
   \[= g(9) = 2(9)^2 + 1 = 163\]

33. a. \(x \mid (f + g)(x)\)
   \begin{align*}
   0 & \quad -7 + 1 = -6 \\
   1 & \quad -4 + 3 = -1 \\
   2 & \quad -1 + 9 = 8 \\
   3 & \quad 2 + 19 = 21
   \end{align*}

   b. \(x \mid (f - g)(x)\)
   \begin{align*}
   0 & \quad -7 - 1 = -8 \\
   1 & \quad -4 - 3 = -7 \\
   2 & \quad -1 - 9 = -10 \\
   3 & \quad 2 - 19 = -17
   \end{align*}

   c. \(x \mid (f \cdot g)(x)\)
   \begin{align*}
   0 & \quad (-7) \cdot (1) = -7 \\
   1 & \quad (-4) \cdot (3) = -12 \\
   2 & \quad (-1) \cdot (9) = -9 \\
   3 & \quad (2) \cdot (19) = 38
   \end{align*}

   d. \(x \mid \left(\frac{f}{g}\right)(x)\)
   \begin{align*}
   0 & \quad \frac{-7}{-7} = 1 \\
   1 & \quad \frac{-4}{3} = -\frac{4}{3} \\
   2 & \quad \frac{-1}{9} = -\frac{1}{9} \\
   3 & \quad \frac{2}{19} = \frac{2}{19}
   \end{align*}

34. a. \(x \mid (f + g)(x)\)
   \begin{align*}
   -3 & \quad -3 + (-1) = -4 \\
   -1 & \quad -1 + (-1) = -2 \\
   1 & \quad 1 + (-1) = 0 \\
   2 & \quad 2 + (-1) = 1 \\
   3 & \quad 3 + (-1) = 2
   \end{align*}

   \(f + g = \{-3, -4, -1, -2, 0, 1, 2, 3\}\)

   b. \(x \mid (f - g)(x)\)
   \begin{align*}
   -3 & \quad -3 - (-1) = -2 \\
   -1 & \quad -1 - (-1) = 0 \\
   1 & \quad 1 - (-1) = 2 \\
   2 & \quad 2 - (-1) = 3 \\
   3 & \quad 3 - (-1) = 4
   \end{align*}

   \(f - g = \{-3, -2, -1, 0, 1, 2, 3\}\)

   c. \(x \mid (f \cdot g)(x)\)
   \begin{align*}
   -3 & \quad (-3) \cdot (-1) = 3 \\
   -1 & \quad (-1) \cdot (-1) = 1 \\
   1 & \quad (1) \cdot (-1) = -1 \\
   2 & \quad (2) \cdot (-1) = -2 \\
   3 & \quad (3) \cdot (-1) = -3
   \end{align*}

   \(f \cdot g = \{-3, 3, -1, 1, -2, 2, 3\}\)

   d. \(x \mid \left(\frac{f}{g}\right)(x)\)
   \begin{align*}
   -3 & \quad \frac{-3}{-1} = 3 \\
   -1 & \quad \frac{-1}{-1} = 1 \\
   1 & \quad \frac{1}{-1} = -1 \\
   2 & \quad \frac{2}{-1} = -2 \\
   3 & \quad \frac{3}{-1} = -3
   \end{align*}

   \(\frac{f}{g} = \{-3, 3, -1, 1, -2, 2, 3\}\)
35. \[ x \rightarrow g(x) \rightarrow f(g(x)) \quad x \rightarrow f(g(x)) \]

-1 \rightarrow 4 \rightarrow 5  
2 \rightarrow 6 \rightarrow 11  
3 \rightarrow 2 \rightarrow 7  
5 \rightarrow 0 \rightarrow 9

36. \[ (f + g)(x) = \frac{x}{2} + 4 \]

37. a. \[ (f + g)(x) = (4x - 2) + (1 - 2x) = 2x - 1 \]  
   Domain: \((-\infty, +\infty)\) or \(\mathbb{R}\)

b. \[ (f \cdot g)(x) = (4x - 2) \cdot (1 - 2x) = -8x^2 + 8x - 2 \]  
   Domain: \((-\infty, +\infty)\) or \(\mathbb{R}\)

c. \[ (f \circ g)(x) = f(g(x)) = f(1 - 2x) = 4(1 - 2x) - 2 = -8x + 2 \]  
   Domain: \((-\infty, +\infty)\) or \(\mathbb{R}\)

38. a. \[ (f - g)(x) = (4x - 2) - (1 - 2x) = 6x - 3 \]  
   Domain: \((-\infty, +\infty)\) or \(\mathbb{R}\)

b. \[ f(x) = \frac{4x - 2}{1 - 2x} = \frac{2(2x - 1)}{(2x - 1)} = -2 \]  
   Domain: \((-\infty, \frac{1}{2}) \cup (\frac{1}{2}, +\infty)\) or \(\mathbb{R} \setminus \{\frac{1}{2}\}\)

c. \[ (g \circ f)(x) = g(f(x)) = g(4x - 2) = 1 - 2(4x - 2) = -8x + 5 \]  
   Domain: \((-\infty, +\infty)\) or \(\mathbb{R}\)

39. \(f\) and \(g\) are not equal. \(f(3)\) is defined and \(g(3)\) is not defined. (The functions do not have the same domains.)

40. First, replace \(f(x)\) with \(y\).

\[ f(x) = 4x + 3 \]
\[ y = 4x + 3 \]

Next, exchange \(x\) and \(y\), then solve for \(y\).

\[ x = 4y + 3 \]
\[ x - 3 = 4y \]
\[ y = \frac{x - 3}{4} \]

Rewrite the inverse using functional notation.

\[ f^{-1}(x) = \frac{x - 3}{4} \]
\[ (f \circ f^{-1})(x) = f\left(\frac{x - 3}{4}\right) \]
\[ = f\left(\frac{x - 3}{4}\right) = f\left(\frac{x - 3}{4}\right) - 3 \]
\[ = (x - 3) + 3 = x \]

41. a. \(F(100) = 400\)

b. \(V(100) = 1.5(100) = 150\)

c. \(C(x) = V(x) + F(x) = 1.5x + 400\)

d. \(A(x) = \frac{C(x)}{x} = \frac{1.5x + 400}{x}\)
42. a. \(N(40) = 3(40) = 120\); The factory can produce 120 desks in 40 hours.

\[
C(120) = \left(150 - \frac{120}{4}\right)(120) = 14,400; \quad \text{The cost of producing 120 desks is $14,400.}
\]

c. \((C \circ N)(40) = C(N(40)) = C(120) = 14,400; \quad \text{The cost of operating the factory for 40 hours is $14,400.}
\]
d. \((C \circ N)(t) = C(N(t)) = C(3t) = \left(150 - \frac{3t}{4}\right)(3t)
\]
e. There are only 168 hours in a week.

43. a. 5, 9, 13, 17, 21, 25,...

b. 5, 1, -3, -7, -11, -15,...

c. 5, 8, 11, 14, 17, 20,...

d. 5, 8, 11, 14, 17, 20,...

44. a. 3, 6, 12, 24, 48, 96,...

b. 3, -6, 12, -24, 48, -96,...

c. 3, 12, 48, 192, 768, 3072,...

d. 1, 3, 9, 27, 81, 243,...

45. a. 9,11, 13, 15, 17,...

b. 4, 7, 12, 19, 28,...

c. 32, 12, 4, 2,...

d. 3, 10, 17, 24, 31,...

46. \(a_n = a_1 + (n-1)d\)

\[a_{101} = 11 + (101 - 1)3 = 311\]

47. \(a_n = a_1 r^{n-1}\)

\[a_{11} = \left(\frac{1}{64}\right)2^{11-1} = 16\]

48. \(a_n = a_1 + (n-1)d\)

\[300 = -20 + (n-1)8\]

\[320 = (n-1)8\]

\[n - 1 = 40\]

\[n = 41\]

49. \(a_n = a_1 r^{n-1}\)

\[78125 = \frac{1}{15625} 5^{n-1}\]

\[1220703125 = 5^{n-1}\]

\[\ln 1220703125 = \ln 5^{n-1}\]

\[\ln 1220703125 = (n-1)\ln 5\]

\[n = \frac{\ln 1220703125}{\ln 5} + 1 = 14\]

50. a. \(\sum_{i=1}^{6} (3i - 2) = (3(1) - 2) + (3(2) - 2) + (3(3) - 2) + (3(4) - 2) + (3(5) - 2) + (3(6) - 2)\)

\[= 1 + 4 + 7 + 10 + 13 + 16 = 51\]

b. \(\sum_{k=3}^{6} (k^2 - 2k) = \left((3^2 - 2(3)) + (4^2 - 2(4)) + (5^2 - 2(5)) + (6^2 - 2(6)) + (7^2 - 2(7))\right)\)

\[= 3 + 8 + 15 + 24 + 35 = 85\]

c. \(\sum_{j=1}^{3} j^3 = (1)^3 + (2)^3 + (3)^3 + (4)^3 + (5)^3 = 1 + 8 + 27 + 64 + 125 = 225\)

d. \(\sum_{i=1}^{6} 10 = 10 + 10 + 10 + 10 + 10 + 10 = 60\)
51. \[ S = \frac{n}{2} (a_1 + a_n) \]
\[ S_{50} = \frac{50}{2} (23 + 366) = 9,725 \]

52. 
\[ 0.121212\ldots = 0.12 + 0.0012 + 0.000012 + \ldots \]
\[ = 12(0.01) + 12(0.01)^2 + 12(0.01)^3 + \ldots \]

\[ S = \frac{a_1}{1-r} = \frac{0.12}{1-0.01} = \frac{4}{33} \]

53. Since \(|r| < 1\) we may use the following formula to evaluate the infinite geometric series.
\[ S = \frac{a_1}{1-r} \]
\[ S = \frac{\frac{3}{5}}{1-\frac{3}{5}} = \frac{3}{2} \]

54. \[ S = \frac{n}{2} (a_1 + a_n) \]
\[ S_{17} = \frac{17}{2} (46 + 80) = 1,071 \text{ cm} \]

55. The perimeter of the third square is \(4(10) = 40 \text{ cm}\). The total perimeter of all the squares is
\[ 4(20) + 4(10\sqrt{2}) + 4(10) + \ldots = 4 \left( 20 + 10\sqrt{2} + 10 \ldots \right) =
4 \left( 20 + 20 \frac{\sqrt{2}}{2} + 20 \left( \frac{\sqrt{2}}{2} \right)^2 + 20 \left( \frac{\sqrt{2}}{2} \right)^3 + \ldots \right) =
\]
\[ S = 4 \left( \frac{a_1}{1-r} \right) = 4 \left( \frac{20}{1-\frac{\sqrt{2}}{2}} \right) = 80 \sqrt{2} + 2 = 160 + 80\sqrt{2} \text{ cm} \]

56. Distance:
\[ d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \]
\[ = \sqrt{(1-(-4))^2 + (-5-7)^2} \]
\[ = \sqrt{(5)^2 + (12)^2} = \sqrt{25+144} = \sqrt{169} = 13 \]

57. Midpoint:
\[ \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{-5 + 7}{2}, \frac{2 + 12}{2} \right) = \left( \frac{2}{2}, \frac{14}{2} \right) = (1, 7) \]

58. To find the center we use the midpoint formula with the given points:
\[ \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{1+5}{2}, \frac{-3+(-3)}{2} \right) = \left( \frac{6}{2}, \frac{-6}{2} \right) = (3, -3) \]

59. The radius is 2 (half the distance between the given points).

60. Substitute: \((h, k) = (3, -3)\) and \(r = 2\) in the standard form of the equation of a circle.
\[ (x-h)^2 + (y-k)^2 = r^2 \]
\[ (x-3)^2 + (y-(-3))^2 = 2^2 \]
\[ (x-3)^2 + (y+3)^2 = 4 \]

61. The equation \(y = 2x - 4\) is in the slope intercept form of a line. Thus the graph is C.

62. The equation \(y = 2(x-1)^2\) is in the standard form of a parabola. Thus the graph is D.

63. The equation \((x-1)^2 + (y+2)^2 = 4\) is in the standard form of a circle. Thus the graph is A.

64. The equation \(\frac{(x-1)^2}{9} + \frac{(y+2)^2}{4} = 1\) is in the standard form of an ellipse. Thus the graph is F.
65. The equation \( \frac{(x-1)^2}{9} - \frac{(y+2)^2}{4} = 1 \) is in the standard form of a hyperbola. Thus the graph is E.

66. The equation \( y = \sqrt{4 - x^2} \) is the upper half of the circle \( x^2 + y^2 = 4 \). Thus the graph is B.

67. The equation \( (x + 3)^2 + (y - 2)^2 = 9 \) is a circle centered at the origin with a radius of 3.

68. The equation \( x^2 - y = 0 \) is an ellipse.

69. The equation \( (x - 1)^2 + (y - 3)^2 = 4 \) is a circle centered at the origin with a radius of 2.

70. The equation \( x^2 + y^2 = 36 \) is a circle centered at the origin with a radius of 6.

71. The equation \( (x + 3)^2 + (y - 2)^2 = 9 \) is a circle centered at the origin with a radius of 3.

72. The equation \( y = \sqrt{25 - x^2} \) is the upper half of the circle \( x^2 + y^2 = 25 \).
73. The equation \( y = -\sqrt{25-x^2} \) is the lower half of the circle \( x^2 + y^2 = 25 \).

74. The equation \( \frac{x^2}{49} + \frac{y^2}{16} = 1 \) is a horizontally positioned ellipse centered at the origin. The length of the major axis is 14 and the length of the minor axis is 8.

75. The equation \( \frac{(x-1)^2}{4} + \frac{(y+3)^2}{9} = 1 \) is a vertically positioned ellipse centered at the point \((1, -3)\). The length of the major axis is 6 and the length of the minor axis is 4.

76. The equation \( \frac{x^2}{36} - \frac{y^2}{9} = 1 \) is a horizontally positioned hyperbola centered at the origin. The length of the fundamental rectangle is 12 and the height of the fundamental rectangle 6.

77. The solutions to the system of equations are the points of intersection: \((-2, 3)\) and \((2, 3)\).

78.
**Mastery Test for Chapter 12**

1. **a.** Answer: \((-2, 7)\)  
   
   **b.** The last row is a contradiction. Thus there is no solution.

   **c.** Solving for \(x\) in the first equation we obtain: \(x = 6 - 2y\). Thus the general solution of the system is \((6 - 2y, y)\). For \(y = 0, 2, \) and \( y = -1\), we obtain the following particular solutions: \((6, 0), (2, 2),\) and \((8, -1)\).

   **d.**
   
   

2. **a.**

   **b.**

   **c.** Solving for \(x\) in the first equation we obtain: \(x = -\frac{1}{7} z\). Solving for \(y\) in the second equation we obtain: \(y = \frac{5}{7} z\). Thus the general solution of the system is \((-\frac{1}{7} - \frac{1}{7} z, \frac{5}{7} + \frac{5}{7} z, z)\).

   **c.**

   The last row is a contradiction. Thus there is no solution.
3. a. The graph of \( f(x) = |x| - 2 \) is the graph of \( y = |x| \) shifted down 2 units. Thus the graph is A.
   b. The graph of \( f(x) = |x| + 2 \) is the graph of \( y = |x| \) shifted up 2 units. Thus the graph is C.
   c. The graph of \( f(x) = (x - 3)^2 \) is the graph of \( y = x^2 \) shifted right 3 units. Thus the graph is D.
   d. The graph of \( f(x) = (x + 3)^2 \) is the graph of \( y = x^2 \) shifted left 3 units. Thus the graph is B.

4. a. \[ x \quad f(x+3) \]
   b. \[ x \quad f(x) - 2 \]
   c. \[ x \quad f(x-3) \]
   d. \[ x \quad f(x)+2 \]

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<td>22 + 2 = 24</td>
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<tr>
<td>5</td>
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<td>7</td>
<td>59 + 2 = 61</td>
</tr>
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5. a. 
   b. 
   c. 
   d. 

6. a. The graph of \( y = 2f(x) \) is the graph of \( y = f(x) \) stretched vertically by a factor of 2.
   Thus the graph is C.
   b. The graph of \( y = -2f(x) \) is the graph of the reflection of \( y = f(x) \) stretched vertically by a factor of 2.
   Thus the graph is D.
c. The graph of \( y = \frac{1}{2} f(x) \) is the graph of \( y = f(x) \) compressed vertically by a factor of \( \frac{1}{2} \). Thus the graph is A.

d. The graph of \( y = -\frac{1}{2} f(x) \) is the graph of the reflection of \( y = f(x) \) compressed vertically by a factor of \( \frac{1}{2} \). Thus the graph is B.

7. a. \((f + g)(x) = (3x - 5) + (x + 2) = 4x - 3\)

b. \((f - g)(x) = (3x - 5) - (x + 2) = 2x - 7\)

c. \((f \cdot g)(x) = (3x - 5) \cdot (x + 2) = 3x^2 + x - 10\)

d. \(\left(\frac{f}{g}\right)(x) = \frac{3x - 5}{x + 2} \) for \( x \neq 2\)

8. a. \((g \circ f)(5) = g(f(5)) = g(2(5) - 1) = g(9) = 9^2 + 1 = 82\)

b. \((g \circ f)(5) = g(f(5)) = g(5^2 + 1) = g(26) = 2(26) - 1 = 51\)

c. \((f \circ g)(x) = f(g(x)) = f(2x - 1) = (2x - 1)^2 + 1 = 4x^2 - 4x + 2\)

d. \((g \circ f)(x) = g(f(x)) = g(x^2 + 1) = 2(x^2 + 1) - 1 = 2x^2 + 1\)

9. a. 4, 7, 10, 13, 16, 19, ...

b. 4, 12, 36, 108, 324, 972, ...

c. 2, 8, 14, 20, 26, 32, ...

d. 2, 8, 32, 128, 512, 2048, ...

10. a. \(S_n = \frac{n}{2} (a_1 + (n-1)d)\)

\[S_5 = \frac{5}{2} (2(2) + (5-1)5) = 60\]

c. \(S_n = \frac{n}{2} (a_1 + a_n)\)

\[S_{100} = \frac{100}{2} (10 + 21) = 1,550\]

d. \(S_n = \frac{a_1}{1-r} \) for geometric series

\[S_{21} = \frac{6(1-(-2)^{21})}{1-(-2)} = 4,194,306\]

e. \(\sum_{i=4}^{6} (3i^2 + i) = (3(1)^2 + 1) + (3(2)^2 + 2) + (3(3)^2 + 3) + (3(4)^2 + 4) + (3(5)^2 + 5) + (3(6)^2 + 6)\)

\[= 4 + 14 + 30 + 52 + 80 + 114 = 294\]

11. a. \(S = \frac{a_1}{1-r}\)

\[S = \frac{1}{1-\frac{1}{2}} = 2\]

b. \(0.888... = 0.8 + 0.08 + 0.008 + ...\)

\[= 8(0.1) + 8(0.1)^2 + 8(0.1)^3 + ...\]

\[S = \frac{a_1}{1-r} = \frac{0.8}{1-0.1} = \frac{8}{9}\]
d. \[ 0.545454... = 0.54 + 0.0054 + 0.000054 + ... \\
= 54(0.01) + 54(0.01)^2 + 54(0.01)^3 + ... \\
S = \frac{a_1}{1-r} = \frac{0.54}{1-0.1} = \frac{6}{11} \]

12. a. The equation \( y = (x+3)^2 - 4 \) is a parabola shifted left three units and down 4 units.

b. The equation \((x-3)^2 + (y+4)^2 = 16\) is a circle with a radius 4 centered at the point \((3, -4)\).

c. The equation \(\frac{(x-3)^2}{25} + \frac{(y+4)^2}{16} = 1\) is a horizontally positioned ellipse centered at \((3, -4)\) with a major axis of length 10 and minor axis of length 8.

d. The equation \(\frac{x^2}{25} - \frac{y^2}{16} = 1\) is a horizontally positioned hyperbola centered at the origin with a fundamental rectangle with length of 10 and height of 8.

13. a. The graph is a parabola reflected across the \(x\) axis shifted up 4 units. Thus the equation is \(y = -x^2 + 4\).

b. The graph is a circle centered at the point \((0, 5)\) and radius 5. Thus the equation is \((x-h)^2 + (y-k)^2 = r^2\):
\[(x-0)^2 + (y-5)^2 = 5^2\]
\[x^2 + (y-5)^2 = 25\]
c. The graph is a horizontally positioned ellipse centered at the origin with a major axis of length 14 \((a = 7)\) and minor axis of length 10 \((b = 5)\). Thus the equation is
\[
\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1
\]
\[
\frac{(x-0)^2}{7^2} + \frac{(y-0)^2}{5^2} = 1
\]
\[
\frac{x^2}{49} + \frac{y^2}{25} = 1
\]

d. The graph is a horizontally positioned hyperbola centered at the origin. It has a fundamental rectangle with a length of 14 \((a = 7)\) and a height of 10 \((b = 5)\). Thus the equation is
\[
\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1
\]
\[
\frac{(x-0)^2}{7^2} - \frac{(y-0)^2}{5^2} = 1
\]
\[
\frac{x^2}{49} - \frac{y^2}{25} = 1
\]