2.2 The Derivative as a Function

Up to this point, we have considered the derivative of a function $f$ at a fixed number $a$:

$$f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

We now change our point of view and allow $a$ to vary. Replacing $a$ by a variable $x$, we obtain

provided the limit exists. Thus, we can regard $f'$ as a new function, called the ________________.

Geometric Interpretation:

The domain of $f'$ is the set \{x|$f'(x)$ exists\} and may be smaller than the domain of $f$.

Example 1. a) If $f(x) = x^3 - x^2 - 2x$, find a formula for $f'(x)$. 
b) Compare the graphs of \( f \) and \( f' \) shown below.

Example 2. The graph of a function \( f \) is given below.

a) Estimate the value of each derivative.

(i) \( f'(1) \)  
(ii) \( f'(3) \)  
(iii) \( f'(6) \)  
(iv) \( f'(7) \)

b) Use the graph of \( f \) to sketch the graph of the derivative \( f' \).
Example 3. Find $f'$ if $f(x) = \frac{1}{x}$. State the domain of $f'$.

Example 4. If $f(x) = \sqrt{2x-3}$, find the derivative of $f$.

Let $y = f(x)$ be a function. Then some common alternative notations for the derivative are:

The symbols $D$ and $d/dx$ are called ___________________________ because they indicate the operation of ___________________________, which is the process of calculating a derivative.
A function \( f \) is \( \quad \) if \( f'(a) \) exists. It is \( \quad \) \( (a, b) \) [or \( (a, \infty) \) or \( (-\infty, a) \) or \( (-\infty, \infty) \)] if it is differentiable at every number in the interval.

Example 5. Where is the function \( f(x) = |x| \) differentiable?
If $f$ is ______________ at $a$, then $f$ is ______________ at $a$.

Note: The converse is false! There are functions that are continuous but not differentiable. What is an example of a function that is continuous, but not differentiable?

**Question:** How can a function fail to be differentiable?

**Answer:**

(1) ____________________________________________________________________________

(2) ____________________________________________________________________________

(3) ____________________________________________________________________________