2.4 Derivatives of Trigonometric Functions

Recall that when we talk about the function $f(x) = \sin x$, it is understood that $\sin x$ means the sine of the angle whose radian measure is $x$.

**Two Special Trigonometric Limits:**

\[
\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = \quad \lim_{\theta \to 0} \frac{\cos \theta - 1}{\theta} =
\]

Proof:
Example 1. Find the limit.

a) \( \lim_{x \to 0} \frac{\sin 4x}{\sin 6x} \)

b) \( \lim_{\theta \to 0} \frac{\cos \theta - 1}{\sin \theta} \)

c) \( \lim_{x \to 1} \frac{\sin(x - 1)}{x^2 + x - 2} \)
Example 2. Let \( f(x) = \sin x \). Use the definition of the derivative to find \( f'(x) \).

Thus, \( \frac{d}{dx} \sin x = \) ____________. Using similar methods, we could find that \( \frac{d}{dx} \cos x = \) ____________.

Example 3. Let \( f(x) = \tan x \). Find \( f'(x) \).
We can find the derivatives of the remaining trigonometric functions using similar methods.

| Derivatives of Trigonometric Functions: |

**Example 4.** Differentiate the functions. Simplify as much as possible.

a) $f(x) = \sqrt{x} \sin x$

b) $g(t) = 4 \sec t + \tan t$
c) $y = \sin \theta \cos \theta$

d) $y = 1 - \sec x \cot x$

Trigonometric functions are often used in modeling real-world phenomena: vibrations, waves, elastic motions, etc.

**Example 5.** An object at the end of a vertical spring is stretched 8 cm beyond its rest position and released at time $t = 0$. Its position at time $t$ is

$$s = f(t) = 8 \cos t.$$  

Find the velocity and acceleration at time $t$ and use them to analyze the motion of the object.
Example 6. Find $\frac{d}{dx} (x \sin x)$. 