2.5 The Chain Rule

Motivating Example: Suppose that Ana is able to roll out 2 times as many cinnamon rolls in an hour as Beth and Beth is able to roll out 3 times as many cinnamon rolls as Cindy. How many more cinnamon rolls can Ana roll out than Cindy?

Goal: Find the derivative of \( h(t) = \sqrt[3]{1 - t^2} \).

Notice that everything we have learned up to this point does not tell us what to do in this situation! This is too difficult given our current set of tools! Up to this point, we have considered fairly simple functions. We will now introduce a powerful tool that will help us differentiate more complicated functions. In reality, this tool is used in calculating every derivative, but because the functions we have looked at so far have been simple, the effects of this tool has been negligible.

Notice that \( h(t) \) is really a composition of functions: \( F(t) = f \circ g(t) \) where

- ______________________
- ______________________

Each of those functions are easy to differentiate using the Differentiation Formulas

- ______________________
- ______________________

Now for the Chain Rule:

If \( g \) is differentiable at \( x \) and \( f \) is differentiable at \( g(x) \), then the composite function \( F = f \circ g \) is defined by \( F(x) = f(g(x)) \) is differentiable at \( x \) and \( F' \) is given by the product

In Leibniz notation, if \( y = f(u) \) and \( u = g(x) \) are both differentiable functions, then
Example 1. Find the derivative of $h(t) = \sqrt[3]{1-t^2}$.

Note: The Chain Rule says that we differentiate the outer function $f$ at the inner function $g$ and then we multiply it by the derivative of the inner function.

Example 2. Differentiate.

a) $F(x) = (4x - x^2)^{100}$

b) $y = 3 \cot n\theta$
c) \( y = x \sin \frac{1}{x} \)

d) \( y = \sqrt{x} + \sqrt{x} + \sqrt{x} \)

e) \( G(y) = \frac{(y - 1)^4}{(y^2 + 2y)^5} \)
Example 3. Find the $x$-coordinates of all points on the curve $y = \sin 2x - 2 \sin x$ at which the tangent line is horizontal.

Example 4. If $h(x) = \sqrt{4 + 3f(x)}$, where $f(1) = 7$ and $f'(1) = 4$, find $h'(1)$. 
