Exam II In-Class Review

1. Find the derivative of the following functions. Do not simplify

a) \( f(x) = x^5 + \sqrt{x} - \frac{3}{x^2} \)

b) \( f(t) = \frac{1 + t^2 - \sqrt[3]{t}}{t^2} \)

c) \( g(x) = \frac{x^2 + x - 4}{2x - x^3} \)

d) \( y = \sec x - 5 \tan x \)

e) \( f(x) = (x^3 + x + 1)^8 \)

f) \( f(x) = \sqrt{x^5 + \frac{3}{x^2} + \sin x - \csc x} \)

g) \( g(x) = \cos^3(x^2 + 9) \)
h) \( h(x) = \frac{x}{(x^5 + 1)^4} \)

i) \( f(x) = |x^2 - 2x| \)

2. Find \( \frac{dy}{dx} \) if \( x^4 - 4x^2y^2 + y^3 = 0 \)

3. Find the 98th derivative of \( f(x) = \frac{1}{x^2} \)
4. Find $f'(\frac{\pi}{6})$ for $f(x) = -2 \cot x$.

5. Using differentials or a linear approximation, approximate $\sqrt{11}$.

6. One side of a right triangle is known to be 20 cm long and the opposite angle is measured as 30°, with a possible error of ±1°.
   a) Use differentials to approximate the maximum error in computing the length of the hypotenuse.
   
   b) What is the percentage error?
7. A particle is moving according to the equation of motion $s(t) = t^4 - 4t + 1$, where $t$ is measured in seconds and $s(t)$ is measured in feet. What is the acceleration of the particle at the instant when the particle is at rest?

8. Let $f(x) = (1 + x^2)^{\frac{3}{2}}$. Find $f''(x)$. Simplify completely.

9. Find the points on the curve $y = x^3 - x^2 - x + 1$ where the tangent lines are horizontal, if any. If there are none, support your answer.
10. Find the following limits:

a) \[ \lim_{x \to 0} \frac{\sin 3x}{5x} \]

b) \[ \lim_{x \to 0} \frac{\sin^2 6x}{x^2} \]

c) \[ \lim_{x \to 0} \frac{\cos x - 1}{\sin x} \]

d) \[ \lim_{x \to 0} \frac{4\cos x - 4 + 3\sin x}{5x} \]
11. Find the equation of the line normal to the curve given by \( y + xy = 8 \) at the point \((-5, -2)\).

12. Find the points on the curve \( y = 8x^3 + 5x + 1 \) where the tangent line has slope 1, if any. If there are none, support your answer.

13. At what point on the curve \( y = x\sqrt{x} \) is the tangent line parallel to the line \( 3x - y + 6 = 0 \)?
14. Let \( f(x) \) be a differentiable function and let \( g(x) = 3x^2 - 1 \). Let \( H(x) = f(g(x)) \), the composite of \( f \) and \( g \). If \( f(0) = 1 \), \( f'(0) = -1 \), \( f(1) = 3 \), \( f'(1) = 2 \), \( f(2) = -1 \), \( f'(2) = 5 \), find \( H'(1) \).

15. Find the linear approximation for \( f(x) = \frac{1}{x} \) at \( x = 1 \).
16. A trough is 20 feet long. The end of the trough is an isosceles triangle with height 10 feet and length of 3 feet across the top. If water is poured in the trough at a rate of 3 cubic feet per minute, how fast is the water level rising when the height of the water is 1 foot?

17. A camera is positioned 4000 feet from the base of a rocket launching pad. At a particular moment, the rocket rises vertically and its speed is 300 ft/s when it has risen 3000 ft. How fast is the distance from the camera to the rocket changing at that moment?
18. If \( f(x) = \frac{1}{x} \), verify \( f(x) \) satisfies the Mean Value Theorem on the interval \([1, 10]\) and find all \( c \) that satisfies the conclusion of the Mean Value Theorem.

19. Find the absolute maximum and minimum of \( f(x) = x^3 - 5x^2 + 3 \) on the interval \([-1, 3]\).
20. The graph of $f$ is given. Use the graph to determine all critical values of $f$, local extrema, and absolute extrema.