Exam IV Review

1. Find the derivative of the function.
   
   a) \( F(x) = \int_x^1 \sqrt{t + \sin t} \, dt \)
   
   b) \( g(x) = \int_1^{\sin x} \frac{1 - t^2}{1 + t^4} \, dt \)
   
   c) \( f(x) = \int_{\ln x}^{2x} e^{-t^2} \, dt \)
2. Evaluate the integral.
   a) \( \int \sec^2 x + e^x \, dx \)
   
   b) \( \int \frac{1}{x} - \sin x \, dx \)
   
   c) \( \int_0^T (x^4 - 8x + 7) \, dx \)
   
   d) \( \int_0^1 (1 - x)^9 \, dx \)
e) \( \int_{0}^{\sqrt{3}} y^3 \sqrt{1 + y^2} \, dy \)

f) \( \int \frac{x + 2}{\sqrt{x^2 + 4x}} \, dx \)

g) \( \int \frac{\cos(\ln x)}{x} \, dx \)

h) \( \int_{0}^{4} |\sqrt{x} - 1| \, dx \)
3. Find the area of the region bounded by the given curves.
   a) \( y = \sqrt{x}, \ y = x^2 \)

   b) \( x + y = 0, \ x = y^2 + 3y \)

   c) \( y = e^x, \ y = e^{-x}, \ x = -2, \ \text{and} \ x = 1 \) (Just set up the integral.)
4. Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.

a) $y = x^2, y = 8 - x^2$; about $y = -1$

b) $y = x^2 + 1, y = 0, x = 0, x = 1$; about the $y$-axis
5. The base of a solid is the region bounded by the circle with radius 1. Find the volume of the solid if the cross sections perpendicular to the $x$-axis are equilateral triangles with one side lying along the base.

6. A 1600-lb elevator is suspended by a 200-ft cable that weighs 10 lb/ft. How much work is required to raise the elevator from the basement to the third floor, a distance of 30 ft?
7. A tank is full of water. Find the work required to pump the water out of the spout. Use the fact that water weighs 62.5 lb/ft$^3$.

8. Find the average value of the function $f(x) = \frac{1}{x}$ on the interval $[1, 4]$.

9. Determine whether $g(x) = x^3 - x$ is one-to-one.
10. The graph of \( g \) is given.

a) Why is \( g \) one-to-one?

b) Estimate the value of \( g^{-1}(2) \).

c) Estimate the domain of \( g^{-1} \).

d) Sketch the graph of \( g^{-1} \).
11. Find the inverse function of \( f(x) = \frac{x + 1}{2x + 1} \).

12. The function \( f(x) = x^3 + 7x - 1 \) is one-to-one. Find \((f^{-1})'(7)\).

13. Sketch the basic graphs of \( f(x) = e^x \) and \( g(x) = \ln x \).
14. Find the following limits:

   a) \[ \lim_{x \to 5^+} e^{x/(5-x)} \]

   b) \[ \lim_{x \to \infty} \frac{1 + e^{2x}}{1 - e^{2x}} \]

   c) \[ \lim_{x \to 2^+} \ln(x - 2) \]

   d) \[ \lim_{x \to \infty} \left[ \log_2(2x^2 - 1) - \log_2(3x^2 + 6) \right] \]
15. Evaluate:
   a) $\frac{1}{2} \log_5 100 - \log_5 14 + \log_5 35$

   b) $\ln \sqrt{e^3}$

16. What is the domain of each of the following functions?
   a) $f(x) = \ln(x^2 + 2x - 8)$

   b) $f(x) = \ln x + \ln(3 - x)$
17. Solve for $x$:
   a) $\log_{10}(x + 3) + \log_{10}(x) = 1$
   
   b) $\ln x - \ln(x + 1) = \ln 2 + \ln 3$

18. Express $\log_8 x - \log_8 \sqrt{9x + 2} + 5 \log_8 (x + 1)$ as a single logarithm.

19. Find the inverse of $f(x) = e^{6x-3}$. 
20. Differentiate each function:
   a) \( f(t) = \cos^2 t \ln t \)
   b) \( f(x) = \ln(\sqrt{x^2 + 1}) \)
   c) \( f(x) = \log_3(\sec x) \)
   d) \( f(x) = 3^{1/x} \)
21. Using logarithmic differentiation, find the derivative of

a) \( y = (\cos x)^{\tan x} \)

b) \( y = \sqrt{x^x} \)

c) \( y = \frac{(x^2 + 2)^2}{(x^4 + 4)^4} \)