1. Evaluate the following integrals.

   a) (3 points) \( \int \left( 3e^t - \frac{1}{5t} \right) dt \)

   \begin{align*}
   \text{Ans: } & 3e^t - \frac{1}{5} \ln |t| + C
   \end{align*}

   b) (5 points) \( \int (4x^3 \sin x^4) \, dx \)

   \begin{align*}
   \text{Ans: } & -\cos x^4 + C
   \end{align*}

   c) (5 points) \( \int_{0}^{1} \frac{x - \sqrt{x}}{3} \, dx \)

   \begin{align*}
   \text{Ans: } & -\frac{1}{12}
   \end{align*}

2. (3 points) Find the domain of \( f(x) = \log_5 \sqrt{x - 4} \). Write your answer in interval notation.

   \begin{align*}
   \text{Ans: } & D = (4, \infty)
   \end{align*}

3. (3 points each) Differentiate each function. You must simplify your answer.

   a) \( f(t) = 8^{2t} \)

   \begin{align*}
   \text{Ans: } & f'(t) = 2 \ln 8 \cdot 8^{2t}
   \end{align*}

   b) \( y = \frac{\ln t}{t} \)

   \begin{align*}
   \text{Ans: } & y' = \frac{1 - \ln t}{t^2}
   \end{align*}
4. (4 points) Make a rough sketch of the graph of \( f(x) = 2 + \frac{1}{2}(4)^{-x} \) using a basic graph and transformations.

\[ Ans: \]

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5. (3 points) Find the exact value of \( \log_3 45 - \log_3 10 + \log_3 18. \)

\[ Ans: 4 \]

6. (5 points) Solve \( \ln x + \ln(x - 1) = 1 \) for \( x. \)

\[ Ans: x = \frac{1 + \sqrt{1 + 4e}}{2} \) (negative option is not in the domain) \]

7. (3 points) Find the derivative of \( F(x) = \int_{\pi/2}^{x^3} \cos t \, dt. \)

\[ Ans: F'(x) = 3x^2 \cos x^3 \]

8. (4 points) Differentiate \( y = (1 + x)^{(x - 2)}. \) Do not simplify.

\[ Ans: y' = \frac{x - 2}{1 + x} + \ln(1 + x) \quad (1 + x)^{(x - 2)} \]

9. (4 points) Find the limit: \( \lim_{x \to 3^+} \ln(x - 3) \)

\[ Ans: -\infty \]
10. (6 points) A certain spring has a natural length of 6 in. A force of 10 lb is required to hold it 5 inches beyond its natural length. How much work is done in stretching the spring from 8 in to 12 in?

Ans: $\frac{8}{3}$ ft-lb (Don’t forget to convert!)

11. (5 points) Let $f(x) = x^3 + 4x - 4$. Find $(f^{-1})'(1)$.

Ans: $\frac{1}{7}$

12. (5 points) The linear density in a rod 12 m long is

$$\rho(x) = \frac{18}{\sqrt{x+4}} \text{ kg/m},$$

where $x$ is measured in meters from one end of the rod. Find the average density of the rod.

Ans: $\bar{\rho} = 6 \text{ kg/m}$

13. (6 points) Given that a particle has velocity given by $v(t) = \frac{1}{2}t^2 - t - 4 \text{ m/s}$, find the distance traveled by the particle during the first 6 seconds.

Ans: distance $= \int_0^6 |v(t)| \, dt = \frac{62}{3} \text{ m}$

14. (7 points) Find the area of the region enclosed by $y = x^2 + 2x + 1$ and $y = 3x + 3$.

Ans: $\frac{9}{2}$

15. Consider $f(x) = \ln(x+3)$.

(a) (2 points) Show that $f(x)$ is one-to-one.

Ans: $f'(x) = \frac{1}{x+3}$ which is always positive on the domain of $f$.
Thus $f$ is always increasing. Thus $f$ is one-to-one

(b) (4 points) Find a formula for $f^{-1}(x)$.

Ans: $f^{-1}(x) = e^x - 3$
16. (8 points) Find the volume of the solid obtained by rotating the region under the curve 
\[ y = \frac{1}{\sqrt{x+1}} \]
from \( x = 0 \) to \( x = 3 \) about the \( x \)-axis using the disk or washer method.

\[ \text{Ans: } \pi \ln 4 \]

17. (7 points) Find the volume of the solid whose base is bounded by the graphs of \( y = 1 - x^2 \) and \( y = 0 \) with cross sections taken perpendicular to the \( x \)-axis are squares.

\[ \text{Ans: } \frac{16}{15} \]

18. (5 points) Use the method of cylindrical shells to set up the integral that would find the volume generated by rotating the region in the first quadrant bounded by \( y = \frac{10}{x^2} \), \( y = 0 \), \( x = 1 \), and \( x = 5 \) about \( x = 10 \). Do not evaluate your integral.

\[ \text{Ans: } V = 2\pi \int_1^5 (10 - x) \frac{10}{x^2} \, dx \]