Math 211: Calculus I

Quiz 5: (3.3-3.5)

Directions: This is closed notes, closed book assignment, meaning you may not get help from anyone or any resource, including the instructor. You may not use a calculator. You must show all of your work. Your work must be organized and legible. You may lose points for missing or illegible work.

1. Let \( f(x) = 36x + 3x^2 - 2x^3 \).
   
a) Find the intervals of increase or decrease. Find the \( x \)-coordinates of the local maxima and minima.

\[
\begin{align*}
  f'(x) &= 36 + 6x - 6x^2 \\
  f'(x) &= 0 \Rightarrow 36 + 6x - 6x^2 = 0 \\
  &\Rightarrow -6(x^2 - x - 6) = 0 \\
  &\Rightarrow -6(x - 3)(x + 2) = 0 \\
  x &= 3, -2
\end{align*}
\]

\( f'(x) \) is polynomial \( \Rightarrow f'(x) \) never undefined

\[
\begin{array}{c|c|c}
  x & f'(x) & f(x) \\
  \hline
  -2 & - & \text{decreasing on } (-\infty, -2) \cup (-2, 3) \cup (3, \infty) \\
  3 & + & \text{has a local maximum at } x = 3 \\
  -2 & + & \text{has a local minimum at } x = -2
\end{array}
\]

b) Find the intervals of concavity and the \( x \)-coordinates of the inflection points.

\[
\begin{align*}
  f''(x) &= 6 - 12x \\
  f''(x) &= 0 \Rightarrow 6 - 12x = 0 \\
  &\Rightarrow 12x = 6 \\
  &\Rightarrow x = 1/2
\end{align*}
\]

\( f''(x) \) is polynomial \( \Rightarrow f''(x) \) never undefined

\[
\begin{array}{c|c|c}
  x & f''(x) & f(x) \\
  \hline
  1/2 & + & \text{concave up on } (-\infty, 1/2) \\
  1/2 & - & \text{concave down on } (1/2, \infty) \\
  \end{array}
\]

f has an inflection point at \( x = 1/2 \)

2. Find \( \lim_{x \to -\infty} \frac{\sqrt{9x^5} - x}{x^3 + 1} \).

\[
\begin{align*}
  \lim_{x \to -\infty} \frac{\sqrt{9x^5} - x}{x^3 + 1} &= \lim_{x \to -\infty} \frac{\frac{1}{x^5} \left( \sqrt{9} - \frac{1}{x^5} \right)}{1 + \frac{1}{x^3}} \\
  &= -\frac{3}{1} = \boxed{-3}
\end{align*}
\]

[Continued on the back.]
3. Sketch the graph of a function satisfying the following conditions:

- Domain: \((-\infty, 1) \cup (1, \infty)\)
- Intercept at \((0, 0)\)
- Horizontal asymptote at \(y = 0\)
- Vertical asymptote at \(x = 1\)
- \(f'(x) > 0\) for \(x < -\frac{1}{2}\)
- \(f'(x) < 0\) for \(-\frac{1}{2} < x < 1\) and \(x > 1\)
- \(f''(x) > 0\) for \(x < -2\) and \(x > 1\)
- \(f''(x) < 0\) for \(-2 < x < 1\)