

Please email me if you have questions or find errors.

FINAL REVIEW EXERCISES

Note: These problems address major concepts in Math 070, but do not comprise all possible problems/concepts covered on the final exam.

1. Find the domain of each function. Express your answer using interval notation.

- a) $f(x) = 7x + 3$ $(-\infty, \infty)$
- b) $f(x) = \sqrt{6 - 2x}$ $(-\infty, 3]$
- c) $f(x) = \frac{x-2}{3x-4}$ $(-\infty, \frac{4}{3}) \cup (\frac{4}{3}, \infty)$
- d) $f(x) = 2^{x-8}$ $(-\infty, \infty)$
- e) $h(x) = \log_2(x-5)$ $(5, \infty)$
- f) $f(x) = 4x^2 - 2x + 3$ $(-\infty, \infty)$

2. Let $f(x) = -2x^2 + 3x$, $g(x) = \frac{2}{x-5}$, $h(x) = \frac{2x}{x+1}$ and $k(x) = 2x - 3$.

Find the given new functions and completely simplify your answers.

- a) $(f \circ g)(x)$
- b) $g(f(-2)) = -\frac{2}{19}$
- c) $h(k(x)) = \frac{2x-3}{x-1}$

3. Find the inverse the given functions.

- a) $f(x) = 4x - 5$ $f^{-1}(x) = \frac{x+5}{4}$
- b) $f(x) = \sqrt[3]{x-2}$ $f^{-1}(x) = x^3 + 2$
- c) $h(x) = \frac{3x}{x+2}$ $h^{-1}(x) = -\frac{2x}{x-3}$

4. a) Stuart has 1000 yd of fencing and wishes to enclose a rectangular area.

What are the dimensions of the rectangle that will maximize the enclosed area? $250 \text{ yd} \times 250 \text{ yd}$
 What is the maximum area that can be enclosed by the fence? $62,500 \text{ sq. yd.}$

b) A farmer uses 1200ft of fence to enclose three sides of a rectangular region.

What are the dimensions of the rectangle that will maximize the enclosed area? $300' \times 600'$
 What is the maximum area that can be enclosed by the fence? $180,000 \text{ sq. ft.}$

c) The larger of two numbers is 8 more than twice the smaller number.

Find the numbers such that their product is a minimum.

Function: $P(x) = 2x^2 + 8x$ where $x =$ the smaller number
 The 2 numbers are -2 and 4 .

5. Simplify the radical expressions and assume that the variables represent real numbers. Use absolute values when necessary.

a) $\sqrt{16x^{12}y^{10}z^6} = 4x^6|y^5z^3|$

b) $\sqrt[3]{40x^4y^6} = 2xy^2\sqrt[3]{5x}$

c) $\sqrt{243a^4b^9c^{13}} = 9a^2b^4c^6\sqrt{3bc}$

d) $\sqrt{\frac{9x^2}{16x^{-2}}} = \frac{3}{4}x^2$

e) $\sqrt[4]{\frac{32}{y^5}} = \frac{2\sqrt[4]{2}}{y\sqrt[4]{y}} \cdot \frac{\sqrt[4]{y^3}}{\sqrt[4]{y^3}} = \frac{2\sqrt[4]{2y^3}}{y^2}$

↑
remember to rationalize the denominator

2a)
 $\frac{2(3x-19)}{(x-5)^2}$

Function: $A(x) = -x^2 + 500x$ where $x =$ width of the rectangle

Function: $A(x) = -2x^2 + 1200x$ where $x =$ width of the rectangle

MATH 070 APPLIED

6. Perform the indicated operations. Assume that the variables represent positive numbers.

a) $2\sqrt[3]{3x^3} + 4x\sqrt[3]{81} - \sqrt[3]{24x^3} = 12x\sqrt[3]{3}$ b) $\sqrt{27x^2} - x\sqrt{48} + 2\sqrt{75x^2} - x\sqrt{243} = 0$

c) $(-125)^{\frac{2}{3}} = \frac{1}{25}$ d) $\frac{1+\sqrt{5}}{2-\sqrt{5}} = -(7+3\sqrt{5})$ e) $(\sqrt{3}+\sqrt{6})^2 = 9+6\sqrt{2}$

7. Perform indicated operation and write the answer in a + bi form.

a) $(3-2i)^2 = 5-12i$ b) $(2+\sqrt{-16})(-1+\sqrt{-9}) = -14+2i$ c) $\frac{3-2i}{5+2i} = \frac{11}{29} - \frac{16}{29}i$

8. Evaluate.

a) $\log_8(-8)$ undefined b) $\log_5 \frac{1}{125} = -3$ c) $3^{\log_3 15 - \log_3 3} = 5$
 d) $\ln e^{3x} = 3x$ e) $\log_{\sqrt{4}} 16 = 4$ f) $\log_2 \frac{1}{16} = -4$ g) $\log_4 100 - \log_4 25 = 1$

9. Write the expression as a sum and/or a difference of multiples of logarithms.

a) $\ln\left(\frac{ex^2}{y\sqrt{z}}\right) = 1 + 2\ln x - \ln y - \frac{1}{2}\ln z$ b) $\log_2\left(\frac{\sqrt[3]{x}}{y^2}\right)^6 = 2\log_2 x - 3\log_2 y$
 c) $\log(100x^2z) = 2 + 2\log x + \log z$ d) $\log_5\left(\frac{yz}{125x^3}\right) = \log_5 y + \log_5 z - 3 - 3\log_5 x$

10. Write the expression as a single logarithm with coefficient 1.

a) $\frac{1}{2}\log x - \log y + 3\log(x-3) = \log\left[\frac{\sqrt{x}(x-3)^3}{y}\right]$
 b) $\log_2(x^2 - 5x + 6) - \log_2(x^2 - 4) + \log_2(x+2) = \log_2(x-3)$
 c) $9\ln\sqrt[3]{x} + 8\ln\sqrt{x} - \ln x = \ln x^6$

11. Solve. Remember to check your answers for possible extraneous solutions, when necessary. Write your answer using set notation.

a) $|x-4|+7=18 \quad \{-7, 15\}$ b) $4|3-x|=4 \quad \{2, 4\}$
 c) $-5|x-2|=20 \quad \emptyset$ d) $|x-3|=|2x-5| \quad \left\{2, \frac{8}{3}\right\}$
 e) $\sqrt{x+2}+\sqrt{x-1}=3 \quad \{2\}$ f) $4\sqrt{x-5}+9=29 \quad \{30\}$
 g) $x^2+4x-6=0 \quad \{-2-\sqrt{10}, -2+\sqrt{10}\}$ h) $2^{x-2}=7^x \quad \left\{\frac{2\ln 2}{\ln 2 - \ln 7}\right\}$

MATH 070 APPLIED

11. Solve. Remember to check your answers for possible extraneous solutions, when necessary. Write your answer using set notation.

i) $\log(x+18) - \log 5 = \log(2x)$ $\{2\}$ j) $\log_2(x+1) + \log_2(x-1) = 3$ $\{3\}$

k) $\left(\frac{1}{4}\right)^{2x} \cdot 2^{6x} = 32$ $\left\{\frac{5}{2}\right\}$

12. Solve using matrices (row operations). Write your answer as an ordered triple.

$$\begin{cases} x + y + z = -2 \\ x + y - 2z = 1 \\ 4x + 2y + 3z = -15 \end{cases} \quad (-5, 4, -1)$$

13. Use Cramer's Rule (determinants) to solve the system. Write your answer as an ordered pair.

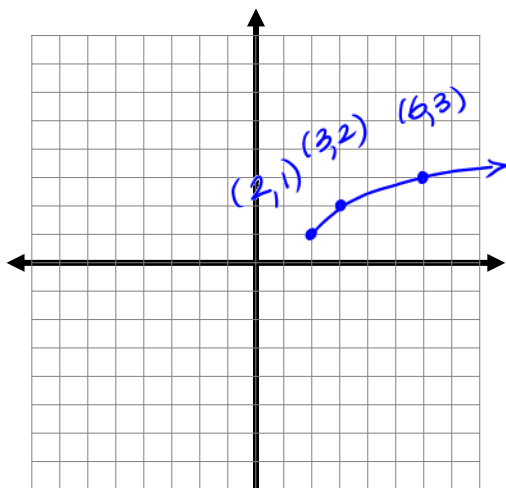
$$\begin{cases} x - y = -2 \\ 5x + 3y = -8 \end{cases} \quad \left(-\frac{7}{4}, \frac{1}{4}\right)$$

14. Solve the inequalities and write your solution set using interval notation:

$(-\infty, -3] \cup (3, \infty)$

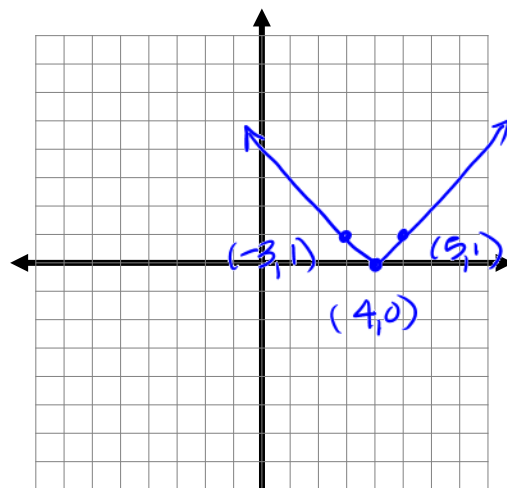
a) $x - 9 > -2x$ or $-2x \geq 6$ b) $2|x - 5| \geq 10$ $(-\infty, 0] \cup [10, \infty)$
 c) $-2|2x + 1| \geq -4$ $\left[-\frac{3}{2}, \frac{1}{2}\right]$ d) $x^2 + 3x - 10 > 0$ $(-\infty, -5) \cup (2, \infty)$
 e) $\frac{x - 5}{x + 2} \geq 0$ $(-\infty, -2) \cup [5, \infty)$ f) $x^2 + 6x + 9 \leq 0$ $\{-3\}$

15. Graph $f(x) = \sqrt{x-2} + 1$ and state the domain and the range of the function.



Domain: $[2, \infty)$
 Range: $[1, \infty)$

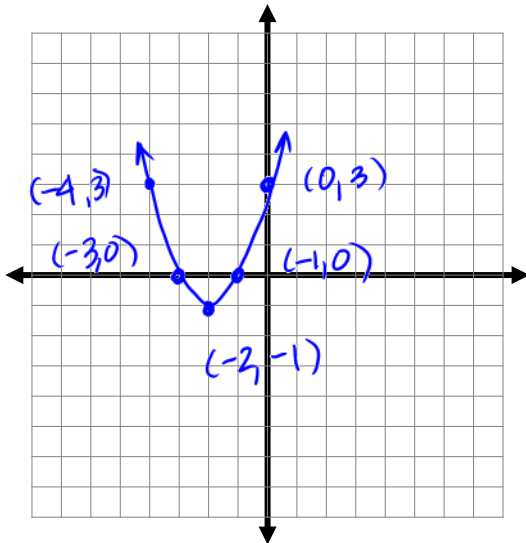
16. Graph $g(x) = |x - 4|$ and state the domain and the range of the function.



Domain: $(-\infty, \infty)$
 Range: $[0, \infty)$

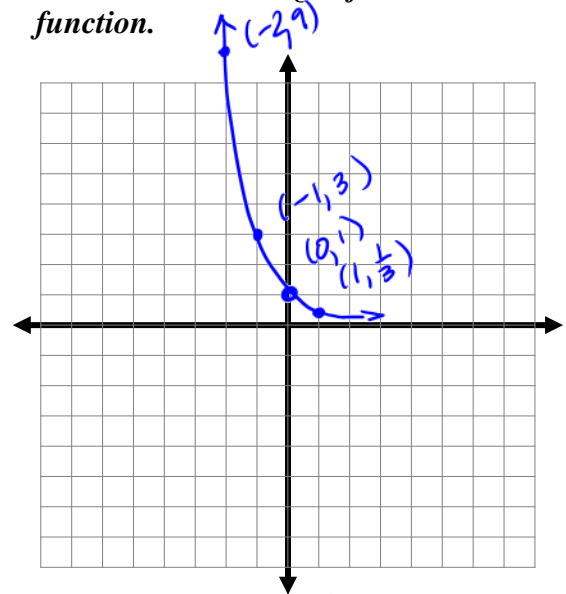
MATH 070 APPLIED

17. Graph $h(x) = x^2 + 4x + 3$ and state the domain and the range of the function.



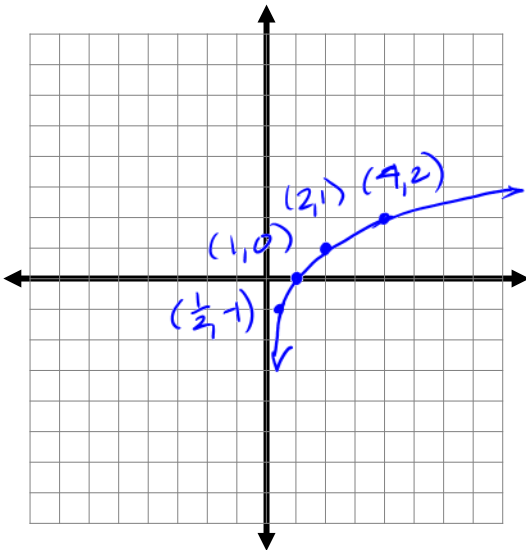
Domain $(-\infty, \infty)$
Range $[-1, \infty)$

18. Graph $k(x) = \left(\frac{1}{3}\right)^x$ and state the domain and the range of the function.



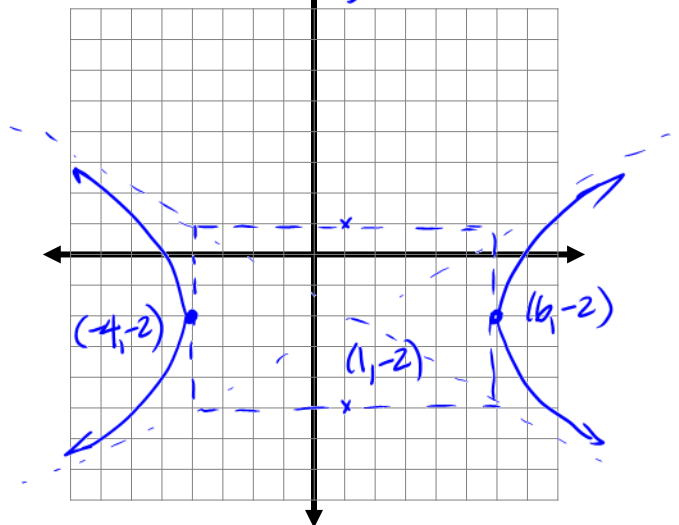
Domain: $(-\infty, \infty)$
Range: $(0, \infty)$

19. Graph $f(x) = \log_2 x$ and state the domain and the range of the function.



Domain: $(0, \infty)$
Range: $(-\infty, \infty)$

20. Graph $9(x-1)^2 - 25(y+2)^2 = 225$
 $\frac{(x-1)^2}{25} - \frac{(y+2)^2}{9} = 1$ Hyperbola.

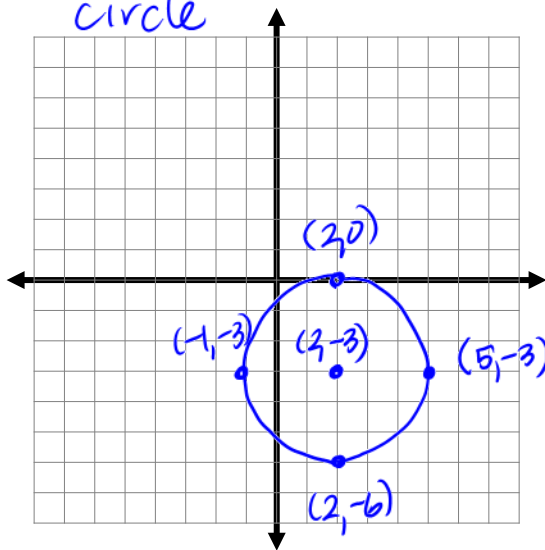


MATH 070 APPLIED

21. Graph $x^2 - 4x + y^2 + 6y + 4 = 0$

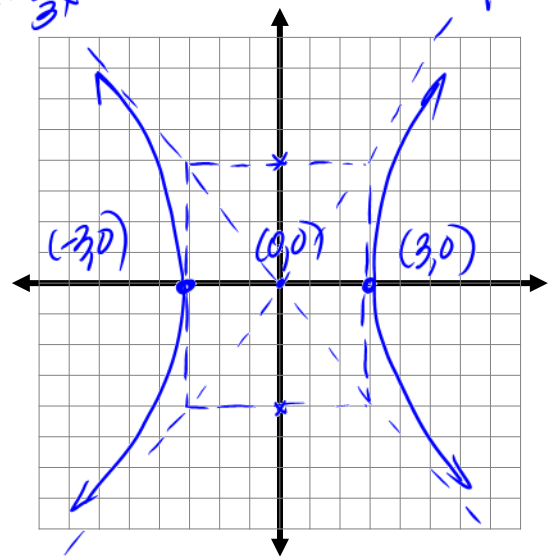
$$(x-2)^2 + (y+3)^2 = 9$$

circle



22. Graph $16x^2 - 9y^2 = 144$ and write the equations of the asymptotes.

$$y = -\frac{4}{3}x \quad y = \frac{4}{3}x$$



Sequences and Series

23. Write a formula for the n th term (a_n) of the sequence:

a) 17, 11, 5, -1, ...

$$a_n = -6n + 23$$

b) $-\frac{1}{2}, \frac{2}{3}, -\frac{3}{4}, \frac{4}{5}, \dots$

$$a_n = (-1)^n \left(\frac{n}{n+1}\right)$$

c) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \dots$

$$a_n = \frac{n}{n+1}$$

d) 2, -4, 8, -16, 32, -64, ...

$$a_n = (-1)^{n+1} 2^n$$

24. Write the given series in summation notation.

a) $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \frac{1}{36} = \sum_{i=1}^6 \frac{1}{i^2}$

b) $1 - 8 + 27 - 64 + 125 = \sum_{i=1}^5 (-1)^{i+1} (i^3)$

c) $30 + 26 + 22 + 18 + 14 + 10 = \sum_{i=1}^6 (-4i + 34)$

d) $\frac{1}{1} - \frac{2}{3} + \frac{4}{9} - \frac{8}{27} + \frac{16}{81} - \frac{32}{243} = \sum_{i=1}^6 (-1)^{i-1} \left(\frac{2}{3}\right)^{i-1} = \sum_{i=1}^6 \left(-\frac{2}{3}\right)^{i-1}$