**College of the Canyons** **Name:** Solutions

**A. Morrow** **Math 214 Quiz 12 – 8.2, 8.3, 8.4** **Spring 2008**

This quiz is closed book. Answer the following questions NEATLY. Show all necessary work directly on the quiz. Answers without supporting work shown will receive no credit.

1) Let T: $P_1$ -> $P_2$ by T(a + bx) = (a-b) + (a-b)x^2.

a) Find a basis for ker(T) *(6 points)*

$(a-b)+(a-b)x^2=0$

$\Rightarrow a-b=0 \Rightarrow a=b.$

$\Rightarrow a+ax \Rightarrow a(1+x)$

Basis: $\{1+x\}$

b) Find a basis for R(T) *(4 points)*

$(a-b)+(a-b)x^2$

$=(a-b)(1+x^2)$

Basis: $\{1+x^2\}$

c) rank(T) = 1 *(1 point)*

d) nullity(T) = 1 *(1 point)*

e) Is T 1-1? Yes/No: NO Why/why not? *(3 points)*

$\text{Ker}(T) \neq \{0\}$

f) Find matrix for T with respect to the bases B = {1, x} and B' = {1, x, 1 + x^2}. *(6 points)*

$T(1) = 1 + x^2 \qquad T(x) = -1 + (-1)x^2 = -1(1+x^2)$

$[T]_{B',B}:\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & -1 \end{bmatrix}$

2) Complete the definitions. If $T: V \to W$ is a linear transformation, then the *(3 points each)*

a) Kernel of T is

$\{v \mid T(v) = 0\}$

b) Range of T is

$\{T(v) \mid v \in V\}$.

3) Complete the statement of the Dimension Theorem for Linear Transformations: If $T: V \to W$ is a linear transformation from an n-dimensional vector space V to a vector space W, then *(3 pts)*

$\text{rank}(T)$ + $\text{nullity}(T)$ = $n$

4) Prove if $T: U \to V$ is a one-to-one linear transformation and $\{\mathbf{u}_1, \ldots, \mathbf{u}_n\}$ is linearly independent in U, then $\{T(\mathbf{u}_1), \ldots, T(\mathbf{u}_n)\}$ is linearly independent in V. *(6 points)*

Given: $T: U \to V$ 1-1, linear ; $\{u_1, \ldots, u_n\}$ LI

Prove: $\{T(u_1), \ldots, T(u_n)\}$ LI

Proof:

let $c_1T(u_1) + \cdots + c_nT(u_n) = 0$.

$\Rightarrow T(c_1u_1 + \cdots + c_nu_n) = 0$ since T linear

$\Rightarrow c_1u_1 + \cdots + c_nu_n = 0$ since T 1-1

$\Rightarrow c_1 = \cdots = c_n = 0$ since $\{u_1, \ldots, u_n\}$ LI

$\Rightarrow \{T(u_1), \ldots, T(u_n)\}$ LI

**Bonus**: Worth 3 extra points for a correct answer. No partial credit.

5) Prove that determinant is a similarity invariant.

G: $B = P^{-1}AP$.

P: $\det(B) = \det(A)$.

Proof:

$$\det(B) = \det(P^{-1}AP) = \det(P^{-1})\det(A)\det(P)$$
$$= \frac{1}{\det(P)} \cdot \det(A)\det(P) = \det(A)$$