College of the Canyons **Name:** SOLUTIONS
A. Morrow **Math 214 Quiz 2 – 1.1-1.5** **Spring 2008**

This quiz is closed book. Answer the following questions NEATLY. Show all necessary work directly on the quiz. Answers without supporting work shown will receive no credit.

1) Complete the definition: If $AB = BA = I$, then A is invertible and B is an inverse of A. *(3 points)* only 1 -2

2) Let A be a square nxn matrix. Then A is invertible is equivalent to (make 3 equivalent statements): *(3 points)*

   a) $Ax = 0$ has only trivial sol^n

   b) $RREF(A) = I$

   c) $A = E_1 \cdots E_k$, E_i elementary

3) Consider the matrices below and find the following. If it is not possible, state such. *(6 points each)*

$$A = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ -1 & 2 & 3 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 3 & 3 \\ -1 & 2 & 3 \end{bmatrix}$$

a) AA^T

$$\begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & -1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 0 \\ 2 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & -2 \\ -2 & 2 \end{bmatrix}$$

b) B^{-1}

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ -1 & 2 & 3 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 5 & 1 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 5 & 1 & -1 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & \frac{1}{5} & -\frac{1}{5} & \frac{1}{5} \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{3}{5} & \frac{7}{5} & -\frac{2}{5} \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & \frac{1}{5} & -\frac{1}{5} & \frac{1}{5} \end{array}\right]$$

$$B^{-1} = \begin{bmatrix} \frac{3}{5} & \frac{7}{5} & -\frac{2}{5} \\ 0 & 1 & 0 \\ \frac{1}{5} & -\frac{1}{5} & \frac{1}{5} \end{bmatrix}$$

c) Find the elementary matrix E such that EB = C.

$R_3 + R_2 \rightarrow R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

4) A square matrix A is symmetric if $A^T = A$. Show that if B is a square matrix, then BB^T is symmetric. *(6 points)*

① Given: B is square

Prove: BB^T symmetric

Proof:

$$(BB^T)^T = (B^T)^T B^T = B B^T$$

$\therefore$ BB^T is symmetric.

5) Prove: If A and B are nxn matrices, then tr(A + B) = tr(A) + tr(B). *(6 points)*

① Given: A, B nxn

Prove: tr (A+B) = tr(A) + tr (B).

Proof:

$$tr(A+B) = \sum_{i=1}^{n} [A+B]_{ii} = \sum_{i=1}^{n} (a_{ii} + b_{ii})$$

$$= \sum_{i=1}^{n} a_{ii} + \sum_{i=1}^{n} b_{ii}$$

$$= tr(A) + tr(B)$$

Bonus: Worth 3 extra points for a correct answer. No partial credit.
Is the sum of two invertible matrices necessarily invertible? Either prove or give a counterexample.

<u>NO</u>

$A = [1]$ $\quad B = [-1]$ both invertible

$A + B = [0]$, not invertible.