College of the Canyons **Name:** Solutions
A. Morrow **Math 214 Quiz 3 – 1.6-2.1** **Spring 2008**

This quiz is closed book. Answer the following questions NEATLY. Show all necessary work directly on the quiz. Answers without supporting work shown will receive no credit.
(3 pts each part unless otherwise noted)

1) Complete the definition: A square matrix A is called symmetric if $A^T = A$.

2) Write a 3x3 matrix that is an example of:

a) A diagonal matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

b) An upper triangular matrix that is not diagonal

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

3) Complete the theorem statement: If A is an invertible matrix, then $A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A)$.

4) Find conditions that the b's must satisfy for the system to be consistent.

$$\begin{cases} x_1 + 2x_2 - x_3 = b_1 \\ -x_2 + 3x_3 = b_2 \\ 2x_1 + 4x_2 - 2x_3 = b_3 \end{cases}$$

$$\begin{bmatrix} 1 & 2 & -1 & b_1 \\ 0 & -1 & 3 & b_2 \\ 2 & 4 & -2 & b_3 \end{bmatrix}$$

$-2R_1 + R_2 \to R_2$

$$\to \begin{bmatrix} 1 & 2 & -1 & b_1 \\ 0 & -1 & 3 & b_2 \\ 0 & 0 & 0 & b_3 - 2b_1 \end{bmatrix}$$

$$\Rightarrow \boxed{b_3 = 2b_1} \Rightarrow \vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ 2b_1 \end{bmatrix}$$

5) Let $A = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. Find the following.

a) M_{23} *(1 point)*

$$\det\begin{pmatrix} 1 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = 0$$

b) C_{23} *(1 point)*

$$(-1)^{2+3} \cdot 0 = 0$$

c) adj(A) *(4 points)*

$$\begin{bmatrix} -9 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ \boxed{18} & 0 & 0 & -9 \end{bmatrix}^T$$

$$(-1)^5 \begin{vmatrix} 0 & 0 & 2 \\ 3 & 0 & 0 \\ 0 & -3 & 0 \end{vmatrix}$$

$$\begin{bmatrix} -9 & 0 & 0 & 18 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & -9 \end{bmatrix}$$

6) Let $A = [\, a_{ij} \,]$ be an nxn matrix. Determine whether A is symmetric when $a_{ij} = i + j$. Justify your answer.

$$a_{ij} = i + j = j + i = a_{ji}$$

Yes Symmetric

7) Let A**x** = **o** be a homogeneous system of n linear equations in n unknowns that has only the trivial solution. Show that if k is any positive integer, then the system A^k**x** = **o** also has only the trivial solution. *(6 points)*

Given: $Ax=0$ has only trivial solⁿ, A $n \times n$, $k \in \mathbb{Z}^+$

Prove: $A^k x = 0$ has only trivial solⁿ.

Proof:

$A\vec{x} = \vec{0}$ only triv. solⁿ.

$\Leftrightarrow$ A invertible

$\Rightarrow$ A^k invertible

$\Leftrightarrow$ $A^k x = 0$ has only triv. solⁿ.

8) Prove: If $A^TA = A$, then A is symmetric and $A = A^2$. *(6 points)*

Given: $A^TA = A$

Prove: A is symmetric and $A = A^2$

Proof:

Assume invertible -3

To show symmetric: $A^T = (A^TA)^T = A^T(A^T)^T$

$= A^T A = A$.

To show $A^2 = A$: $A^2 = A \cdot A = A^T \cdot A = A$

Bonus: Worth 3 extra points for a correct answer. No partial credit.
True or False: If A + B is symmetric, then so are A and B. Either prove or give a counterexample.

$A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ $A+B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$