College of the Canyons **Name:** Solutions
A. Morrow **Math 214 Quiz 4 – 2.2-2.4** **Spring 2008**

This quiz is closed book. Answer the following questions NEATLY. Show all necessary work directly on the quiz. Answers without supporting work shown will receive no credit.

1) Fill in the Blank. Matrix Properties. A, B, C invertible nxn matrices. k scalar. *(1 point each)*

a) $(AB)^T =$ $B^T A^T$

b) $(AB)^{-1} =$ $B^{-1}A^{-1}$

c) $(A^T)^{-1} =$ $(A^{-1})^T$

d) $A(B - C) =$ $AB - AC$

e) $(A^T)^T =$ A

f) $(A + B)^T =$ $A^T + B^T$

g) $\det(AB) =$ $\det(A)\det(B)$

h) $\det(A^{-1}) =$ $\frac{1}{\det(A)}$

i) $\det(kA) =$ $k^n \det(A)$

j) $\det(A^T) =$ $\det(A)$

2) Let A be a square nxn matrix. Then A is invertible is equivalent to (make 1 equivalent statement that pertains to the determinant): *(3 points)*

a) $\det(A) \neq 0$

3) Consider (1, 4, 2, 3), a permutation of {1, 2, 3, 4}. *(1 point each)*

a) Find the number of inversions

$0 + 2 + 0 + 0 = \boxed{2}$

b) Classify the permutation as even or odd.

even

4) Given that $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 2$, find $\begin{vmatrix} -a & -b & -c \\ 4d & 4e & 4f \\ g-d & h-e & \cancel{f-i} \end{vmatrix}$. *(6 points)*

$i-f$

$2(-1)(4) = \boxed{-8}$

5) For the system $\begin{cases} x_1 + 2x_2 = \lambda x_1 \\ 2x_1 + x_2 = \lambda x_2 \end{cases}$, find the following: *(3 points each)*

a) Characteristic equation

missing det -1

$$\det \begin{pmatrix} \lambda-1 & -2 \\ -2 & \lambda-1 \end{pmatrix} = 0$$

b) The eigenvalues

$$\lambda^2 - 2\lambda + 1 - 4 = 0$$

$$\lambda^2 - 2\lambda - 3 = 0$$

$$(\lambda-3)(\lambda+1) = 0$$

$$\lambda = 3, -1$$

c) Pick one eigenvalue. Find the eigenvectors corresponding to that eigenvalue.

$\lambda = 3$:

$$\left[\begin{array}{cc|c} 2 & -2 & 0 \\ -2 & 2 & 0 \end{array}\right]$$

$$\downarrow$$

$$\left[\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$x_1 = x_2$

$$\left\{ t\begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$$

$\lambda = -1$

$$\left[\begin{array}{cc|c} -2 & -2 & 0 \\ -2 & -2 & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$x_1 = -x_2$

$$\left\{ t\begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$$

6) Let A and B be nxn matrices. Show that if A is invertible, then det(B) = det(A^{-1}BA). *(6 points)*

Given: A, B are nxn ; A invertible

Prove: det(B) = det(A^{-1}BA)

Proof:

$$\det(A^{-1}BA) = \det(A^{-1})\det(B)\det(A)$$

$$= \frac{1}{\det(A)}\det(B)\det(A)$$

$$= \det(B)$$

Bonus: Worth 3 extra points for a correct answer. No partial credit.
True or False: The determinant of a matrix is unchanged if the columns are written in the reverse order. Either prove or give a counterexample.

False: $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$\downarrow$

$\det(A) = 1$ $\det(B) = -1$