College of the Canyons **Name:** Solutions
A. Morrow **Math 214 Quiz 5 – 3.1-3.4** **Spring 2008**

This quiz is closed book. Answer the following questions NEATLY. Show all necessary work directly on the quiz. Answers without supporting work shown will receive no credit.

1) Given **u** = (1, 0, -1), **v** = (3, 1, 2). Find the following: *(3 points each)*

a) $||\mathbf{u}||$

$$\boxed{\sqrt{2}}$$

b) $\mathbf{u} \cdot \mathbf{v}$

$$3 + 0 + (-2)$$

$$\boxed{1}$$

c) $\mathbf{u} \times \mathbf{v}$

$$\begin{vmatrix} i & j & k \\ 1 & 0 & -1 \\ 3 & 1 & 2 \end{vmatrix} = i - 5j + k$$

$$\boxed{(1, -5, 1)}$$

+5j (-1)

d) $\text{proj}_{\mathbf{u}}\mathbf{v}$

$$= \frac{u \cdot v}{\|u\|^2} u$$

norm (-2)
slightly wrong formula (-2)

$$= \frac{1}{2}(1, 0, -1)$$

$$= \boxed{\left(\frac{1}{2}, 0, -\frac{1}{2}\right)}$$

e) The component of **v** orthogonal to **u**

$$v - \text{proj}_u v$$

$$(3, 1, 2) - \left(\frac{1}{2}, 0, -\frac{1}{2}\right)$$

$$\boxed{\left(\frac{5}{2}, 1, \frac{5}{2}\right)}$$

f) Cosine of the angle between **u** and **v** (you do not need to find the angle)

$$\cos\theta = \frac{u \cdot v}{\|u\| \|v\|}$$

$$= \frac{1}{\sqrt{2}\sqrt{14}}$$

$$= \boxed{\frac{1}{\sqrt{28}} \text{ or } \frac{1}{2\sqrt{7}}}$$

g) A vector orthogonal to **u** and **v**

$$\boxed{(1, -5, 1)}$$

2) Fill in the blanks with equivalent vector statements to make true vector propoerties. Do not use components. **u**, **v**, **w** ∈ R3. k, l scalars. *(1 point each)*

a) $||k\mathbf{u}||$ = $|k| \, ||u||$

b) $k(\mathbf{u}\cdot\mathbf{v})$ = $(ku)\cdot v = u\cdot(kv)$

c) $\mathbf{u}\times\mathbf{u}$ = $\vec{0}$

3) Prove: If **u** is a vector in 3-space and k and l are scalars, then (k+l)**u** = k**u** + l**u**. *(6 points)*

Given: $u = (u_1, u_2, u_3)$; $k, l \in \mathbb{R}$.

Prove: $(k+l)u = ku + lu$

Proof: $(k+l)u = (k+l)(u_1, u_2, u_3) = ((k+l)u_1, (k+l)u_2, (k+l)u_3)$

$= (ku_1 + lu_1 ; ku_2 + lu_2 ; ku_3 + lu_3)$

$= (ku_1 ; ku_2 ; ku_3) + (lu_1 ; lu_2 ; lu_3)$

$= ku + lu$

4) Prove: **u** = (a, b) and **v** = (-b, a) are orthogonal vectors. *(6 points)*

Given: $u = (a, b)$; $v = (-b, a)$

Prove: u, v are orthogonal

Proof: $u\cdot v = (a,b)\cdot(-b,a) = -ab + ba = 0$

$\therefore$ u, v are orthogonal.

Start w/ u·v

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Bonus: Worth 3 extra points for a correct answer. No partial credit.
True or False: If **u** is orthogonal to **v**+**w**, then **u** is orthogonal to **v** and **w**. Either prove or give a counterexample.

$u = (1,1)$ $v = (1,1)$ $w = (-1,-1)$.

$v + w = (0,0)$

$u\cdot(v+w) = 0$ but $u\cdot v \neq 0$ and $u\cdot w \neq 0$.