Note: Only 35 points. # wrong subtracted from 36 ⇒ inflates score

College of the Canyons Name: Solutions
A. Morrow **Math 214 Quiz 9 – 5.5-6.2 (First Part)** **Spring 2008**

This quiz is closed book. Answer the following questions NEATLY. Show all necessary work directly on the quiz. Answers without supporting work shown will receive no credit.

Complete the definitions (#1-2):

1) An **inner product** on a vector space V is a function that associates a real number $\langle \mathbf{u}, \mathbf{v} \rangle$ with each pair of vectors **u** and **v** in V in such a way that the following axioms are satisfied for all vectors **u**, **v**, and **z** in V and all scalars k: *(1 point each)*

   a) $\langle u, v \rangle = \langle v, u \rangle$

   b) $\langle u+v, z \rangle = \langle u, z \rangle + \langle v, z \rangle$

   c) $\langle ku, v \rangle = k\langle u, v \rangle$

   d) $\langle v, v \rangle \geq 0$ and $\langle v, v \rangle = 0$ only if $v = \vec{0}$

2) Two vectors **u** and **v** in an inner product space are called **orthogonal** if $\langle u, v \rangle = 0$. *(3 points)*

3) Complete the statement of the Dimension Theorem for Matrices: *(3 points)*

   If A is a matrix with n columns, then rank(A) + nullity(A) = n

4) Consider the following set of vectors: $\mathbf{v}_1 = (1, 0, 3, 4, 5)$, $\mathbf{v}_2 = (5, 2, -3, 1, 0)$, $\mathbf{v}_3 = (-2, 1, 0, 0, 3)$.

   a) To find a subset of the vectors that forms a basis for the space spanned by the vectors, write the matrix that you would row reduce. DO NOT REDUCE IT! *(3 points)*

same vectors

$$\begin{bmatrix} 1 & 5 & -2 \\ 0 & 2 & 1 \\ 3 & -3 & 0 \\ 4 & 1 & 0 \\ 5 & 0 & 3 \end{bmatrix}$$

   b) Would your goal be to find the row space, column space, or nullspace for the matrix you just wrote? column space *(1 pt)*

5) Let $\langle \mathbf{p}, \mathbf{q} \rangle = p(0)q(0) + p(1)q(1)$ be an inner product on P_1. Let $\mathbf{p} = x$, and $\mathbf{q} = 1$. Find the following: *(2 points each)*

   a) $\langle \mathbf{p}, \mathbf{q} \rangle$

   $= 0 \cdot 1 + 1 \cdot 1$

   $= 1$

   b) $\|\mathbf{q}\| = [\langle q, q \rangle]^{1/2}$ ①

   $= [1 \cdot 1 + 1 \cdot 1]^{1/2}$

   $= \sqrt{2}$ ①

6) For the matrix $A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, find the following: *(3 points each)*

a) Basis for the row space

$$\left\{ (1,0,1,0),\ (0,1,-1,0),\ (0,0,0,1) \right\}$$

b) Basis for the column space

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

c) Basis for the nullspace

$x_1 = -x_3 = -t$
$x_2 = x_3 = t$
$x_4 = 0$
$x_3 = t$

$$\left\{ (-1,1,1,0) \right\}$$

d) Nullity(A) = 1 *(1 pt)*

e) Rank(A) = 3 *(1 pt)*

7) Prove: If **v** is a vector in a real inner product space, then ⟨**0**, **v**⟩ = 0. *(6 points)*

Given: $\vec{v}$ vector in real inner product space.

Prove: $\langle \vec{0}, \vec{v} \rangle = 0$.

Proof: $0\vec{v} = \vec{0}$

$$\langle \vec{0}, \vec{v} \rangle = \langle 0\vec{v}, \vec{v} \rangle = 0 \langle \vec{v}, \vec{v} \rangle = 0.$$

Use $\mathbb{R}^n$ -2
$n = \#$ -3

Bonus: Worth 3 extra points for a correct answer. No partial credit.
8) Define $W^\perp$.

$= \{ \vec{u} \in V \mid \vec{u}$ orthogonal to $W \}$