College of the Canyons **Name:** Solutions
A. Morrow **Math 214 Test 1A – 1.1-1.4** **Spring 2010**

This quiz is closed book. Answer the following questions NEATLY. Show all necessary work directly on the quiz. Answers without supporting work shown will receive no credit.

Definitions *(3 points each)*

1) True or False: The linear system in question #4 is inconsistent. FALSE

2) Fill in the term: The linear system $Ax = 0$ is called a homogeneous system.

3) Complete the definition: A matrix A is an invertible matrix if __.

there exists a matrix B s.t. $AB = BA = I$.

Procedures *(6 points each part). Show all work.*

4) Solve using Gauss-Jordan elimination: $\begin{cases} x_1 - 2x_2 + x_3 - 3x_4 = 0 \\ 4x_1 - 8x_2 + 4x_3 - 7x_4 = -5 \end{cases}$.

$$\begin{bmatrix} 1 & -2 & 1 & -3 & 0 \\ 4 & -8 & 4 & -7 & -5 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 1 & -3 & 0 \\ 0 & 0 & 0 & 5 & -5 \end{bmatrix}$$

5)

$$\to \begin{bmatrix} 1 & -2 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$x_1 = 2x_2 - x_3 - 3$

$x_4 = -1$

$$\{(2s - t - 3, s, t, -1) : s, t \in \mathbb{R}\}$$

Let $A = \begin{bmatrix} 0 & -1 & 2 \\ -1 & 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 2 & 3 \\ -1 & 0 & 4 \end{bmatrix}$.

a) Find $tr\left(AB^T\right)$.

$$= tr\left(\begin{bmatrix} 0 & -1 & 2 \\ -1 & 0 & 3 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 2 & 0 \\ 3 & 4 \end{bmatrix} \right)$$

$$= tr\left(\begin{bmatrix} 4 & 8 \\ 9 & 13 \end{bmatrix} \right) = 4 + 13 = 17$$

b) Find $-3(A - B)$.

$$= -3 \begin{bmatrix} 0 & -3 & -1 \\ 0 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 9 & 3 \\ 0 & 0 & 3 \end{bmatrix}$$

Proofs *(3 points each)*

6) Let k and l be scalars, and let A be an $m \times n$ matrix. Prove: $(k+l)A = kA + lA$.

$(k+l)A$ is $m \times n$, as is $kA + lA$.
Therefore they are the same size.

Let $[A]_{ij} = a_{ij}$.

$$[(k+l)A]_{ij} = (k+l)a_{ij} = ka_{ij} + la_{ij}$$
$$= [kA]_{ij} + [lA]_{ij} = [kA + lA]_{ij}$$

Therefore $(k+l)A = kA + lA$.

7) Prove: If A and B be invertible matrices, then AB is invertible and $(AB)^{-1} = B^{-1}A^{-1}$.

Let

$$(AB)(B^{-1}A^{-1}) = A(BB^{-1})A^{-1} = AIA^{-1}$$
$$= AA^{-1} = I$$

$$(B^{-1}A^{-1})(AB) = B^{-1}(A^{-1}A)B = B^{-1}IB$$
$$= B^{-1}B = I$$

Therefore, AB is invertible and
$(AB)^{-1} = B^{-1}A^{-1}$.

8) Let A be a $m \times n$ matrix. Prove: $A - A = \vec{0}$, where $\vec{0}$ is the zero matrix.

Let A and $\vec{0}$ be $m \times n$.

Then $A-A$ is $m \times n$ too.

Let $[A]_{ij} = a_{ij}$

Then
$$
\begin{aligned}
[A-A]_{ij} &= [A+(-1)A]_{ij} \\
&= a_{ij} + [(-1)A]_{ij} \\
&= a_{ij} + (-1)a_{ij} \\
&= 0 = [0]_{ij}
\end{aligned}
$$

Therefore $A - A = \vec{0}$

Bonus: *(3 extra points possible, no partial credit)*

Note: A $n \times n$ matrix A is symmetric if $A^T = A$.

If B is $n \times n$ and invertible and B^{-1} is symmetric, is B necessarily symmetric? Explain (prove or give a counterexample).

Yes.

Let B be $n \times n$, invertible. B^{-1} symmetric.

We need to show $B = B^T$.

$$B^{-1}(B^T) = (B^{-1})^T(B^T) = (BB^{-1})^T = I^T = I$$

$$B^T(B^{-1}) = B^T(B^{-1})^T = (B^{-1}B)^T = I^T = I$$

Since the inverse is unique, $B^T = B$.