College of the Canyons **Name:** Solutions
A. Morrow **Math 214 Test 1B – 1.1-1.4** **Spring 2010**

This quiz is closed book. Answer the following questions NEATLY. Show all necessary work directly on the quiz. Answers without supporting work shown will receive no credit.

Definitions *(3 points each)*

1) Complete the definition: A matrix A is an invertible matrix if ___.

there exists a matrix B s.t. $AB = BA = I$.

2) True or False: The linear system in question #4 is consistent. True

3) Fill in the term: The linear system $Ax = 0$ is called a homogeneous system.

Procedures *(6 points each part). Show all work.*

4) Solve using Gauss-Jordan elimination: $\begin{cases} x_1 - 2x_2 + x_3 - 3x_4 = 0 \\ 3x_1 - 6x_2 + 3x_3 - 7x_4 = 4 \end{cases}$.

$$\left[\begin{array}{ccccc} 1 & -2 & 1 & -3 & 0 \\ 3 & -6 & 3 & -7 & 4 \end{array}\right] \rightarrow \left[\begin{array}{ccccc} 1 & -2 & 1 & -3 & 0 \\ 0 & 0 & 0 & 2 & 4 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccccc} 1 & -2 & 1 & 0 & 6 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right]$$

$x_1 = 2x_2 - x_3 + 6$

$x_4 = 2$

$\{(2s - t + 6, s, t, 2) : s, t \in \mathbb{R}\}$.

5) Let $A = \begin{bmatrix} 0 & -1 & 2 \\ -1 & 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & -3 \\ -2 & 4 & 2 \end{bmatrix}$.

a) Find $tr\left(AB^T\right)$. $= tr\left(\begin{bmatrix} 0 & -1 & 2 \\ -1 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 0 & 4 \\ -3 & 2 \end{bmatrix}\right)$

$= tr\left(\begin{bmatrix} -6 & 0 \\ -10 & 8 \end{bmatrix}\right) = -6 + 8 = 2$

b) Find $-2(A - B)$. $= -2\begin{pmatrix} -1 & -1 & 5 \\ 1 & -4 & 1 \end{pmatrix}$

$= \begin{pmatrix} 2 & 2 & -10 \\ -2 & 8 & -2 \end{pmatrix}$

Proofs *(3 points each)*

6) Let k be a scalar, and let A and B be $m\times n$ matrices. Prove: $k(A+B)=kA+kB$.

$k(A+B)$ is $m \times n$, as is $kA+kB$.
Therefore they are the same size.

Let $[A]_{ij} = a_{ij}$. $[B]_{ij} = b_{ij}$

$$[k(A+B)]_{ij} = k[A+B]_{ij} = k(a_{ij}+b_{ij})$$
$$= ka_{ij} + kb_{ij} = [kA]_{ij} + [kB]_{ij}$$
$$= [kA+kB]_{ij}$$

Therefore, $k(A+B) = kA + kB$

7) Prove: If Let A is an invertible matrix, then A^T is invertible and $\left(A^T\right)^{-1}=\left(A^{-1}\right)^T$.

$$(A^T)(A^{-1})^T = (A^{-1} \cdot A)^T = I^T = I$$

$$(A^{-1})^T(A^T) = (A \cdot A^{-1})^T = I^T = I$$

Therefore, A^T is invertible and
$(A^T)^{-1} = (A^{-1})^T$.

8) Let A be a $m \times n$ matrix. Prove: $A + \vec{0} = A$, where $\vec{0}$ is the zero matrix.

Let A and $\vec{0}$ be $m \times n$.

Then $A + \vec{0}$ is $m \times n$ too.

Let $[A]_{ij} = a_{ij}$

$$
\begin{aligned}
\text{Then } [A + \vec{0}]_{ij} &= a_{ij} + 0 \\
&= a_{ij} \\
&= [A]_{ij}.
\end{aligned}
$$

Therefore $A + \vec{0} = A$

Bonus: *(3 extra points possible, no partial credit)*

Note: A $n \times n$ matrix A is symmetric if $A^T = A$.

If B is $n \times n$ and invertible and B^{-1} is symmetric, is B necessarily symmetric? Explain (prove or give a counterexample).

Yes.

Let B be $n \times n$, invertible, B^{-1} symmetric.

We need to show $B = B^T$.

$$B^{-1}(B^T) = (B^{-1})^T(B^T) = (BB^{-1})^T = I^T = I$$

$$B^T(B^{-1}) = B^T(B^{-1})^T = (B^{-1}B)^T = I^T = I$$

Since the inverse is unique, $B^T = B$.