College of the Canyons **Name:** Solutions

A. Morrow **Math 214 Test 3B – 3.1-4.2** **Spring 2010**

This quiz is closed book. Answer the following questions NEATLY. Show all necessary work directly on the quiz. Answers without supporting work shown will receive no credit. *3 points each.*

1) Let $\mathbf{v} = (-4, 0, 3)$ and $\mathbf{u} = (-1, -3, 7)$. Find the following

a) $\|\mathbf{v}\|$

$= \sqrt{16 + 9}$

$= 5$

b) $\operatorname{proj}_{\mathbf{u}} \mathbf{v}$

$= \frac{u \cdot v}{\|u\|^2} u$

$= \frac{25}{59}(-1, -3, 7)$

c) $\mathbf{u} \times \mathbf{v}$

$$\begin{vmatrix} i & j & k \\ -1 & -3 & 7 \\ -4 & 0 & 3 \end{vmatrix} = -9i - 25j - 12k$$

$= (-9, -25, -12)$

d) Find the equation of the line containing $\mathbf{u}$ and $\mathbf{v}$.

$-9x - 25y - 12z = 0$

2) Find the standard matrix for the transformation $T(x, y, z) = (4x + 5y - z, x - y)$.

$$\begin{bmatrix} 4 & 5 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

3) From the past. Fill-in-the-blanks.

If $A\mathbf{x} = \lambda\mathbf{x}$ has nonzero solutions, then λ is a(n) eigenvalue and $\mathbf{x}$ is its corresponding eigenvector.

4) Orthogonal Vectors

a) Complete the definition of orthogonal vectors. Vectors $\mathbf{u}$ and $\mathbf{v}$ are orthogonal if $u \cdot v = 0$.

b) Let $\mathbf{w}$ and $\mathbf{w}_1$ be vectors. Prove that if $\mathbf{w}$ is orthogonal to $\mathbf{w}_1$, then $\mathbf{w}$ is orthogonal to $k\mathbf{w}_1$.

Let $w \perp w_1$. $\therefore$ $w \cdot w_1 = 0$.

$w \cdot (k w_1) = k(w \cdot w_1) = k(0) = 0.$

$\therefore$ $w \perp (k w_1)$

5) Let $\mathbf{u}, \mathbf{v} \in R^3$. Prove $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$.

Let $u = (u_1, u_2, u_3)$, $v = (v_1, v_2, v_3)$

$$u \cdot v = u_1 v_1 + u_2 v_2 + u_3 v_3$$

$$= v_1 u_1 + v_2 u_2 + v_3 u_3$$

$$= v \cdot u$$

6) Unit Vectors and Scalar Multiplication

a) If k s a scalar and $\mathbf{u}$ is a vector, then $\|k\mathbf{u}\| =$ $|k| \, \|\vec{u}\|$.

b) Let $\mathbf{v}$ be a vector. Apply ~~parts~~ part (a) ~~and (b)~~ to show that $\frac{1}{\|\mathbf{v}\|}\mathbf{v}$ is a unit vector.

We must show $\left\| \frac{1}{\|v\|} v \right\| = 1$.

$$\left\| \frac{1}{\|v\|} v \right\| = \left| \frac{1}{\|v\|} \right| \|v\| \quad \text{by part (a).}$$

$$= \frac{1}{\|v\|} \|v\|$$

$$= 1$$

7) State the Cauchy-Schwarz Inequality.

Let $\vec{u}, \vec{v} \in \mathbb{R}^n$.

$|\vec{u} \cdot \vec{v}| \leq \|\vec{u}\| \|\vec{v}\|$

Bonus: *(3 extra points possible, no partial credit)*

A freebie if you took me seriously when I said "memorize"... List the 8 properties of vector arithmetic.

1) $u + v = v + u$

2) $(u + v) + w = u + (v + w)$

3) $u + (-u) = 0$

4) $u + 0 = u$

5) $k(u + v) = ku + kv$

6) $(k + l)u = ku + lv$

7) $(kl)u = k(lu)$.

8) $1u = u$