College of the Canyons **Name:** ~~Solutions~~

A. Morrow **Math 214 Test 7A – 7.3-8.6** **Spring 2010**

This quiz is closed book. Answer the following questions NEATLY. Show all necessary work directly on the quiz. Answers without supporting work shown will receive no credit. *3 points each part unless otherwise marked.*

1) Complete the definition: If T: V → W is a function from a vector space V into a vector space W, then T is a **linear transformation** from V to W if, for all vectors **v**, **u** ∈ V and all scalars c,

$T(u+v) = T(u) + T(v)$

$T(cu) = cT(u)$

2) Complete the statement of the Dimension Theorem for Linear Transformations: If T : V → W is a linear transformation from an n-dimensional vector space V to a vector space W, then

$\text{rank}(T) + \text{nullity}(T) = n$

3) Complete the definitions. If T : V → W is a linear transformation, then the

a) Kernel of T is

$\{\vec{v} \in V : T(\vec{v}) = \vec{0}_W\}$

b) Range of T is

$\{T(\vec{v}) : \vec{v} \in V\}$

4) Find the missing dimensions for the linear transformations below *(1 point each)*

a) T: $M_{22} \to R$ by T(A) = tr(A). nullity(T) = 3

onto. $\text{rank}(T) = 1$

$\dim(V) = 4$

b) T: $R^4 \to R^7$, T is one-to-one. rank(T) = 4

$\text{nullity}(T) = 0$

$\dim(V) = 4$

c) T: V → W, T is an isomorphism between two finite-dimensional vector spaces. dim(W) = 2.

dim(V) = 2

5) Let $T(ax^2 + bx + c) = (a + c,\ b)$.

a) Find a basis for the kernel of T

$T(ax^2+bx+c)=(0,0) \Rightarrow a+c=0,\ b=0.$

$\Rightarrow a=-c.$

Basis: $\{x^2-1\}$

b) Find a basis for the range of T

T onto. $(T(x^2)=(1,0),\ T(x)=(0,1)\}.$

Basis: $\{(1,0),\ (0,1)\}$

c) Find the matrix for T with respect to the bases B = { $x^2 + x$, $x + 1$, $x - 1$ } and B' = { (1,1) , (0,1) }.

$T(x^2+x)=(1,1)$ $\downarrow$ $\begin{bmatrix}1\\0\end{bmatrix}$

$T(x+1)=(1,1)$ $\downarrow$ $\begin{bmatrix}1\\0\end{bmatrix}$

$T(x-1)=(-1,1).$ $\downarrow$ $\begin{bmatrix}-1\\2\end{bmatrix}$

$$\Rightarrow \begin{bmatrix}1 & 1 & -1\\ 0 & 0 & 2\end{bmatrix}$$

6) Let $\mathbf{v}_0$ be a fixed vector in an inner product space V, and let T: V → R be defined by $T(\mathbf{v}) = \langle \mathbf{v}, \mathbf{v}_0 \rangle$. Prove that T is a linear transformation.

Let $\vec{v}, \vec{w} \in V$. $c \in \mathbb{R}$.

① $T(\vec{v}+\vec{w}) = \langle \vec{v}+\vec{w}, \vec{v}_0 \rangle = \langle \vec{v}, \vec{v}_0 \rangle + \langle \vec{w}, \vec{v}_0 \rangle$
$= T(\vec{v}) + T(\vec{w})$

② $T(c\vec{v}) = \langle c\vec{v}, \vec{v}_0 \rangle = c\langle \vec{v}, \vec{v}_0 \rangle$
$= cT(\vec{v})$.

7) Given T: $R^2 \to R^2$ by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 2x_2 \\ 2x_1 \end{bmatrix}$.

a) Is it possible to find an orthonormal basis relative to which the matrix for T is diagonal? Why or why not?

Yes. $[T] = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$ is symmetric.

8) For each transformation below, circle **all** the classifications that apply (there may be more than one): *(1 point each)*

a) T: $R^2 \to R$, $T(x,y) = x$

1-1 ONTO ISOMORPHISM LINEAR NONE OF THESE

b) T: $P_n \to P_n$, $T(p(x)) = p(0) + p(x)$

~~1-1~~ ~~ONTO~~ ISOMORPHISM LINEAR NONE OF THESE

c) T: $V \to R$ by $T(\mathbf{v}) = 1 + ||\mathbf{v}||$, where V is an inner product space.

1-1 ONTO ISOMORPHISM LINEAR NONE OF THESE

9) True/False: *(3 points total... -1.5 for each incorrectly answered, -1 for each not attempted. Max -3)*

a) If $T: V \to U$ is a linear transformation, then R(T) is a subspace of ~~V~~ U . False

b) Let A be an nxn matrix. If A has n real eigenvalues, then A is a symmetric matrix. False

$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

c) If there exists an invertible matrix P such that $B = P^{-1} A P$, then A and B are both invertible. False

$A = B = 0$ $P = I$

Bonus: *(3 extra points possible, no partial credit)*
If A^2 and B^2 are similar, must A and B be similar? If yes, provide a proof. If no, provide a counterexample.

Yes/No? NO

$A = I$, $B = -I$