Chapter 6: Systems of Linear Equations

6.1 Solving Systems of Linear Equations by Graphing

Equations considered together are called a system of equations.

A solution of a system of equations in two variables is an ordered pair that is a solution of each equation of the system.

$2x + y = 3$ $x + y = 1$ $(2, -1)$?

$(3, -3)$?

Is (1,-3) a solution of the system

$$3x + 2y = -3$$
$$x - 3y = 6?$$

Is (-1,-2) a solution of the system

$$2x - 5y = 8$$
$$-x + 3y = -5?$$

The solution of a system of linear equations in two variables can be found by graphing the two equations on the same coordinate system.

Independent

Inconsistent

Dependent

Solve by graphing: $2x + 3y = 6$
$2x + y = -2$

Solve by graphing: $2x - y = 1$
$6x - 3y = 12$

Solve by graphing: $x - 2y = 2$
$x + y = 5$

Solve by graphing: $4x - 2y = 6$
$y = 2x - 3$

Solve by graphing: $x + 3y = 3$
$-x + y = 5$

Solve by graphing: $6x - 2y = -6$
$y = 3x - 1$

6.2 Solving Systems of Linear Equations by Substitution Method

Solve by substitution: $2x + 5y = -11$
$y = 3x - 9$

Solve by substitution: $5x + y = 4$
$y = -5x + 4$

Solve by substitution: $y = 3x - 1$
$y = -2x - 6$

Solve by substitution: $3x - y = 4$
$y = 3x + 2$

Solve by substitution: $7x - y = 4$
$3x + 2y = 9$

Solve by substitution: $y = -2x + 1$
$6x + 3y = 3$

For what value of k does the system have no solution $y = -2x + 1$

$2y = kx + 2$

Rewrite each equation so the coefficients and constant are integers

$.7x - .1y = .4$

$.3x + .2y = .9$

6.3 Solving Systems of Linear Equations by Addition Method

$$3x + 2y = 4$$
$$4x - 2y = 10$$

Sometimes you need to multiply first

$$3x + 2y = 7$$
$$5x - 4y = 19$$

Sometimes you need to multiply both eq.

$$5x + 6y = 3$$
$$2x - 5y = 16$$

$$5x = 2y - 7$$
$$3x + 4y = 1$$

Solve by the addition method:

$$2x + y = 2$$
$$4x + 2y = -5$$

$$2x + 4y = 7$$
$$5x - 3y = -2$$

Solve by the addition method:

$$x - 2y = 1$$
$$2x + 4y = 0$$

$$2x - 3y = 4$$
$$-4x + 6y = -8$$

6.4 Application Problems in Two Variables

Rate-of-wind and water–current problems

- Choose one variable to represent the rate of the object in calm conditions
- Choose second variable to represent rate of wind or current
- Use these variables to express the rate of object with (or without) the wind or current
- Use these variables to express the rate of object against (or with) wind or current
- Create table: Rate x time = distance
- Set each variable expression for distance equal to known distance.
- Solve for one variable
- Substitute and solve for second variable

Flying with the wind, a small plane can fly 750 mi in 3 h. Against the wind, the plane can fly the same distance in 5 h. Find the rate of the plane in calm air and the rate of the wind.

A 600-mile trip from one city to another takes 4 h when a plane is flying with the wind. The return trip against the wind takes 5h. Find the rate of the plane in still air and the rate of the wind.

A canoeist paddling with the current can travel 24 mi in 3 hrs. Against the current, it takes 4 h to travel the same distance. Find the rate of the current and the rate of the canoeist in calm water.

Application Problems

- Choose one variable to represent one unknown quantity
- Choose second variable to represent other unknown quantity
- Write numerical or variable expressions for all remaining quantities
- Record results in two tables
- Determine an equation from the first table
- Determine a second equation from the second table
- Solve for one variable
- Substitute and solve for second variable

A jeweler purchased 5 oz of a gold alloy and 20 oz of a silver alloy for a total cost of $700. The next day, at the same prices per ounce, the jeweler purchased 4 oz of the gold alloy and 30 oz of the silver alloy for a total cost of $630. Find the cost per ounce of the silver alloy.

A storeowner purchased 20 incandescent light bulbs and 30 fluorescent bulbs for a total cost of $40. A second purchase, at the same prices, included 30 incandescent bulbs and 10 fluorescent bulbs for a total cost of $25. Find the cost of an incandescent bulb and of a fluorescent bulb.

Two coin banks contain only dimes and quarters. In the first bank, the total value of the coins is $3.90. In the second bank, there are twice as many dimes as in the first bank and one-half the number of quarters. The total value if the coins in the second bank is $3.30. Find the number of dimes and the number of quarters in the first bank.