11.1 Sequences

An infinite sequence $\{a_n\}$ is a function whose domain is the set of positive integers. The function values, or terms, are represented by $a_1, a_2, a_3, \ldots a_n, \ldots$

List all the terms of

$a_n = -n^2$ for $1 \le n \le 4$

$a_n = \frac{(-1)^n}{n^3}$, the first four terms are?

$a_n = 2n + 5$, the first four terms are?

$a_n = \frac{(-1)^n}{2^n + 1}$, the first four terms are?

Write a formula for the general term of each infinite sequence.

5, 7, 9, 11, 13, . . .

1, -3, 9, -27, . . .

Summation Notation

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_n$$

The indicated sum of the terms of a sequence is called a series.

Find the sum of each series:

$$\sum_{i=3}^{7} \left(i^2 - 4\right) =$$

$$\sum_{i=1}^{6} 2i^2 =$$

$$\sum_{k=3}^{5} \left(2^k - 3\right) =$$

$$\sum_{i=1}^{5} 4 =$$

Write each series in summation notation:

$1^2 + 2^2 + 3^2 + \cdots + 9^2$

$$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^{n-1}}$$

Complete the rewriting of each series using the new index as indicated

$$\sum_{k=1}^{6} k^3 = \sum_{j=0}$$

$$\sum_{k=5}^{10} 2^{-i} = \sum_{j=1}$$

11.2 Arithmetic Sequences

In an arithmetic sequence, each term after the first differs from the preceding term by a constant, the common difference.

General Term of an Arithmetic Sequence

$$a_n = a_1 + (n - 1)d$$

Find the general term and the tenth term:

3, 7, 11, 15, …….

Find the ninth term of the arithmetic sequence whose first term is 6 and whose common difference is –5.

According to the U.S. Bureau of Economic Analysis, U.S. travelers spent $12,808 million in other countries in 1984. This amount has increased by approximately $2350 million yearly. Write a formula for the nth term. How much will U.S. travelers spend in 2010?

Sum of the first n terms of an arithmetic sequence:
The sum, S_n, of the first n terms of an arithmetic sequence is given by

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Find the sum of the first ten terms:

2, 5, 8, 11, ….

Find the sum of the first 15 terms of the arithmetic sequence 3, 6, 9, 12, …

Using S_n to evaluate a summation.

Find the following sum: $\sum_{i=1`}^{30} (6i - 11)$

Nursing home cost is modeled $a_n = 1800n + 49730$ where n is years after 2000. How much would it cost for ten years of nursing home care beginning in 2001?

11.3 – Geometric Sequences

In a geometric sequence, each term after the first is obtained by multiplying the preceding term by a nonzero constant, the common ratio.

Write the first six terms of the geometric sequence with first term 12 and common ratio $\frac{1}{2}$.

General Term of a Geometric Sequence

The nth term (the general term) of a geometric sequence with first term a_1 and common ratio r is

$$a_n = a_1 r^{n-1}$$

Find the seventh term of the geometric sequence whose first term is 5 and whose common ratio is –3.

Write the general term for the geometric sequence

3, 6, 12, 24, 48, …

Find the general term and the ninth term:

$12, -6, 3, -\frac{3}{2}, \ldots$

Sum of the First n Terms of a Geometric Sequence

$$S_n = \frac{a_1\left(1 - r^n\right)}{1 - r}$$

Find the sum of the first nine terms of the geometric sequence: 2, –6, 18, –54, …

Find $\sum_{i=1}^{8} 4 \cdot 3^i$

A job pays a salary of $30,000 the first year. During the next 29 years, the salary increases by 6% each year. What is the total lifetime salary over the 30-year period?

An annuity is a sequence of equal payments made at end of equal time periods. P is regular deposit, r is annual interest rate, interest compounded n times per year, A is value of annuity after t years.

$$A = P\frac{\left(1+\frac{r}{n}\right)^{nt} - 1}{\frac{r}{n}}$$

If \$3000 is deposited into an IRA at the end of each year for 40 years and the interest rate is 10% per year compounded annually, find the value of the IRA after 40 years.

Sum of an Infinite Geometric Series
If –1 < r < 1, the the sum of the infinite geometric series $a_1 + a_1r + a_1r^2 + a_1r^3 + \cdots$

$$S = \frac{a_1}{1-r}$$

If $|r| \geq 1$

Find the sum: $6 + \frac{6}{3} + \frac{6}{3^2} + \frac{6}{3^3} + \cdots$

Find the sum: $3 + 2 + \frac{4}{3} + \frac{8}{9} + \cdots$

Writing a repeating decimal as a fraction.
Express .111…. as a fraction in lowest terms.

11.4 Binomial Theorem

$(a + b)^1 = a + b$
$(a + b)^2 = a^2 + 2ab + b^2$
$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
$(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

What is the 4^{th} term of $(a + b)^8$?

What is the coefficient of the 4^{th} term of $(a + b)^8$?

Factorial Notation
$n! = n(n - 1)(n - 2)(n - 3) \cdots (3)(2)(1)$ and $0! = 1$

Definition of a Binomial Coefficient

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$\binom{8}{3} =$$

$$\binom{6}{3} =$$

$$\binom{6}{0} =$$

$$\binom{8}{2} =$$

$$\binom{3}{3} =$$

A Formula for Expanding Binomials: The Binomial Theorem: For any positive integer n,

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \cdots + \binom{n}{n}b^n$$

Expand $(x + 1)^4 =$

Expand $(3x - y)^4 =$

Finding a Particular Term of a Binomial Expansion

The r^{th} term of the expansion of $(a + b)^n$ is

$$\binom{n}{r-1} a^{n-r+1} b^{r-1}$$

Find the fifth term in the expansion of $(2x + y)^9$?