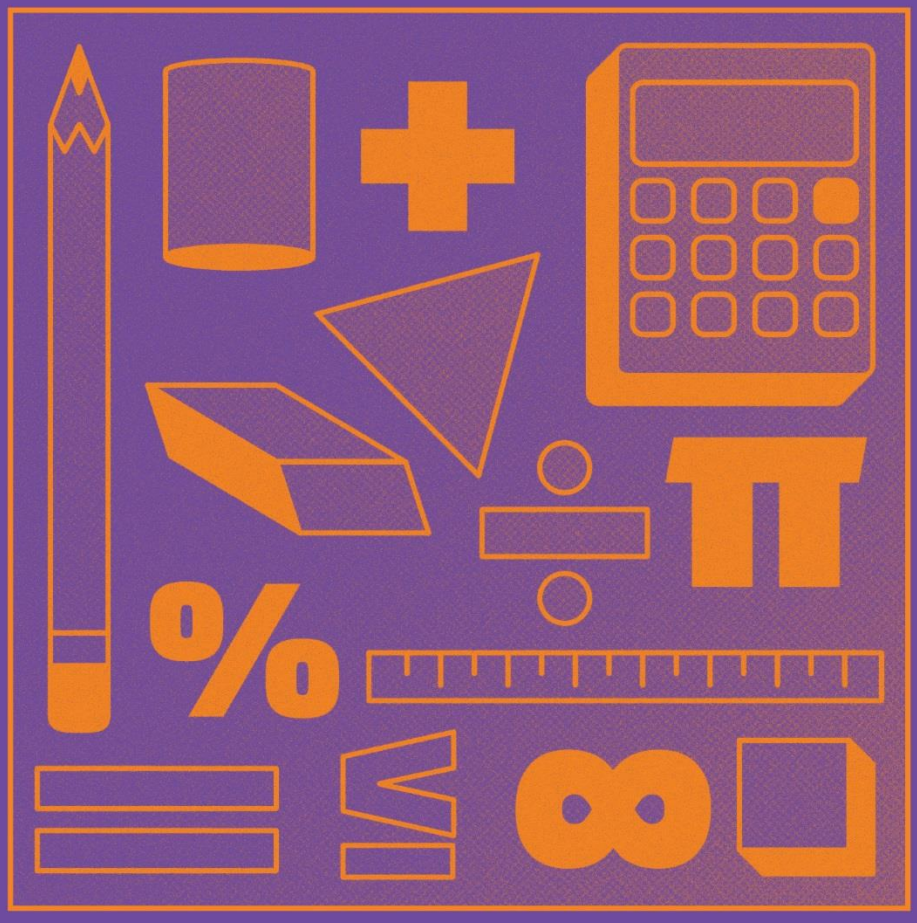
MATH FOR ELEMENTARY SCHOOL TEACHERS



COLLEGE OF THE CANYONS

AMY LAGUSKER, M.A.

MATH 130

Math for Elementary School Teachers

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Authored and Compiled by Amy Lagusker

Editor: Trudi Radtke

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Written & Compiled by:
Amy Lagusker

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Dedicated to my wonderful daughter

Madeline

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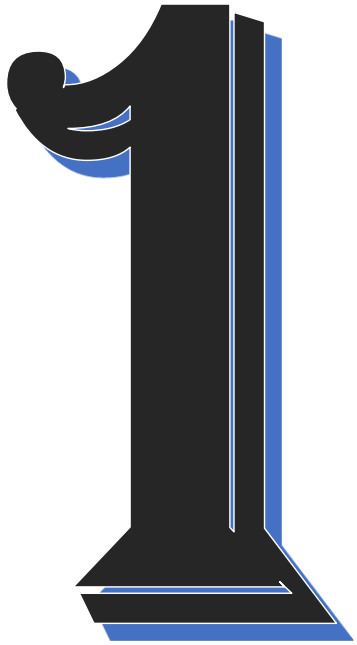


1 - Overview of Elementary Common Core Mathematics

First things, first: How to read this textbook

This introduction chapter gives you a summary of the textbook, a brief history of Mathematics Education, information on Common Core and resources you can use in your future classroom. Use this chapter for your discussions on Canvas and your Lesson Plans for this class. Take the time to read it on your own as it is good background knowledge for when you are a teacher.

Each chapter is broken into Lessons. Most lessons have Partner Activities, which we will complete and discuss in class. After each lesson is the Practice Problems, which you will complete as homework and bring questions to our next meeting. Each chapter begins with The Essential Question(s). We start this way since many textbooks for Elementary schools also start this way. Think of the Essential Question as the Big Idea of the Chapter, the overarching goal of the chapter.

This book is made for you, the future teacher. Write in this book. That is how it is designed. Take notes. Complete your homework here. Keep everything in this book so you have something to take to your future classroom.

Each chapter ends with a Methods of Teaching Extension.

Appendices:

- **A.** The Common Core State Standards
    - **1)** Covers Grades K through 6
    - **2)** You will use this for a few assignments throughout the semester
    - **3)** Do not memorize this enormous document, but rather know how to use it to find information.
- **B.** The Mathematical Practices for Teachers
    - **1)** Shows you how to teach the mathematics
    - **2)** Think of this as the standards for Teachers, guidelines to help you teach well
- **C.** Answers to Partner Activities, use if you are absent
- **D.** Answers to Practice Problems (homework answers, not step-by-step solutions)

Besides this textbook, be sure to complete your assignments in Canvas. Make sure you keep a good pace in Khan Academy. Use your Calendar and Syllabus to make sure you know what we are doing during each class meeting.

The Importance of Mathematics for Elementary School Students

At the beginning of each chapter, we will discuss WHY the mathematics is taught in elementary school and WHY it is important for teachers to understand it, besides we need to understand it since we are going to teach it.

What is Common Core?

A common *Common Core* misconception is that Common Core is a way of teaching. No. Common Core is just a list of standards, which are required to be taught during the duration of the school year. HOW you teach it is up to your principal, vice principal, department chair, your team, etc.

How Did We Get Here?

A Brief History of the Creation and Need for Common Core Mathematics

Somewhere around 2007 to 2008, a few mathematicians got together and decided that students are not prepared for the 21st century. They spoke with physicists, engineers, business people and the like to figure out what the current student needs in this world of having all the information of the entire world in their pocket. Remember when people used to say "You need to learn this because you won't be walking around with a calculator in your pocket"? Those days are gone forever. Today, with the right technology, anyone can solve complex equations (i.e. photo math) graph anything (see graphing apps, as there are many) or even ask google to do it for him or her.

However, what about analytical thinking and problem solving? The original reason why mathematics became a school requirement was to teach students how to think. Technology has not figured out a way to problem solve. That is where Common Core steps in. Yes, we are still teaching the basics. Fractions are not going anywhere, sorry. Nevertheless, we cannot use technology to solve problems. A computer still is stupid. Seriously. It follows the instructions, which comes from a human being. A human being needs to correctly give the instructions to the computer. Common Core challenges students to solve problems conceptually and discover the meaning behind the calculator. For example, think about dividing fractions. A trick that many people have learned is to change the division to multiplication and flip the second term. Then multiply. But why does that work? (We will explore this in Chapter 3.) Another example which Common Core wanted to stress is Mental Math. Make people smarter by forcing them to learn mental math tricks. Think about it, how do you train your body to run a marathon? You run a little bit each day and gradually improve your mileage and then gradually improve your speed. Your body is improving each day that you practice. The brain is the same. Work on

mental math activities a little bit each day, and you are training your brain to be stronger and a better thinking. Math trains you to be smarter in general, which can help you master all other subjects.

The Impact of Common Core in Elementary School Mathematics

When Common Core first arrived it was daunting, from a teacher's and parent's perspective. Photos were floating around social media about bad math techniques. Parents and students disagreed on how to solve a problem. It was joked about in Disney's *Incredibles 3*, where the Dad is trying to help his son do his homework and he becomes very frustrated that "they" changed math. The way subtraction is taught has changed. The way multiplication is taught has changed. The authors wanted students to understand the meaning behind subtraction and multiplication instead of just rote memorization. The biggest change was pushing down the curriculum. What students are learning now in elementary is around two years earlier than students before Common Core. Kindergarten is fully academic, no longer just learning to be social and learning how to play nice.

How has Common Core changed assessments?

The biggest change is having students complete Performance Tasks during their exam. The first part of an exam can be basic questions. The next part could have error analysis (find the mistake). There could be a matching portion as well. There are many methods to access students without having multiple-choice answers or simply asking to solve a math problem without any context to why. Then there are the Performance Tasks, which always show up on State Standardized Testing. A typical Performance Task starts with a situation, a more involved word problem. Students then have to go through a process of steps, answering questions along the way to eventually reach the overall question. There could be solving and graphing involved in the same Performance Task.

How has Common Core changed grading?

Another large change brought by Common Core is how teachers grade assessments. Before, we cared about the final answer. Right or wrong? If wrong, a teacher would look for ways to give partial credit. Now, we look at everything and assign points accordingly. For example, say a student is asked to perform a long division problem worth five points. Getting the right answer is one point out of five. HOW they got to the answer is worth four points. Did they move the decimal correctly? Did they subtract correctly? Did they make a mistake twice and ended up being lucky and getting to the right answer? Did they check their work by multiplying back?

Comparing the Old Traditional Standards to the New Common Core Standards: Kindergarten through 3rd Grade

The major change to 4th through 6th grade are more pre-algebra standards.

Information in the table is pulled from corestandards.org.

Kindergarten	
New: Common Core Standards	**Old: California State Standards**
Count to 100 by ones and 10s.	Count to 30 by ones.
Identify whether the number of objects in one group is greater than, less than or equal to the number of objects in another group.	Compare two or more sets of objects and identify which set is equal to, more than or less than the other.
Solve word problems that require addition and subtraction for problems with sums up to and including 10. Use objects or drawings to represent the problem.	Use objects to determine the answers to addition and subtraction problems.
Break numbers between 11 and 19 into two parts: 10 ones and some further ones. For example, 17 contains 10 ones and seven additional ones.	Not taught until 1st grade.
Put two shapes together to form a different shape. For example, place two triangles together to make a rectangle.	Not taught. New standard.
Moved to first grade.	Tell time and recite the days of the week.

Grade 1	
New: Common Core Standards	**Old: California State Standards**
Solve word problems that call for addition of three numbers whose sum is less than or equal to 20 by using objects, drawings or equations.	Replaces the old standard that required students to "commit to memory" addition equations with a sum of 20 or less and subtraction equations with a difference of 20 or less. Notably, the new standard does not require memorization.
Apply properties of operations (commutative and associative) as strategies to add and subtract.	New to 1st grade. The associative property (a + (b + c) = (a +b) + c) was previously introduced in 2nd grade. The commutative property (a + b = b + a) was not mentioned in the state standards for kindergarten through 3rd grade.
Determine the unknown number in an addition or subtraction equation. For example, 8 + x = 11.	New standard. Previously, students were not expected to find for "x" until after 3rd grade.
Given a two-digit number, mentally find 10 more or 10 less than that number, without having to count. Explain your reasoning.	Similar to the old standard that asked students to identify one more than, one less than, 10 more than and 10 less than a given number.
Tell and write time in hours and half hours using analog and digital clocks.	Previously introduced in kindergarten, although 1st grade students were also required to tell time to the nearest half hour under the old standards.
Eliminates previous 1st grade standards requiring students to understand weight, volume and the monetary value of coins, among other specific skills found in the old 1st	The old standards called for 1st grade students to work with weight, volume, classifying objects by color and size, estimating sums, committing math facts to memory, writing

Grade 1	
grade standards. Also eliminates requirement that students memorize sets of numbers.	number sentences and understanding the value of coins.

Grade 2	
New: Common Core Standards	**Old: California State Standards**
Use addition and subtraction to solve one- and two-step word problems where the sum or difference is less than 100.	New standard. Multi-step word problems were not mentioned in the old standards for kindergarten through 3rd grade.
Easily solve addition and subtraction problems with a sum or difference of less than 20 in your head (7 - 4 = 3; 2 + 9 = 11; 14 + 3 = 17; etc.). By end of grade 2, memorize all the ways to add two one-digit numbers.	Similar to the old standard that required students to find the sum or difference of two, two-digit numbers in their heads (14 + 16 = 30; 12 + 5 = 17; 32 - 7 = 25). However, the old standard required students to add and subtract larger numbers.
Use addition to find the total number of objects arranged in equal rows. For example, if there are three rows of four, students should be able to add 4+4+4 to find the total, rather than counting each object. Students must also write an equation to represent how they found the total number of objects.	Replaces the old standards that focused on multiplication and division, which required students to do "repeated" addition and subtraction (2 + 2 + 2 + 2 = 8), form equal groups from a set of objects (sort eight blocks into four groups of two) and know the multiplication tables for twos, fives and 10s.
Explain strategies that can be used to make addition and subtraction easier. For example, for the problem 14 - 5 = x, a student might know that 15 - 5 = 10. Because 14 is one less than 15, the student could figure out that answer to the problem presented would also therefore be one less, or 14 - 5 = 9. Students	New standard. The old standards did not call for student explanations of specific math processes, nor did they call for specific instruction in developing strategies to add and subtract faster. Instead, a standard at the end of each grade level called for students to be able to "justify their reasoning" in general.

Grade 2	
should be able to use these strategies and explain why they work.	
Understand that each number falls a set distance from zero on a number line. Use a diagram of a number line to find sums and differences of less than 100. For example, for the problem 82 - 17 = x, start at 82 on the number line and count down 17 spaces to determine that 82 - 17 = 65.	New standard. Common Core's focus on teaching children about the number line comes from research showing that familiarity with number lines improves mathematical performance in young children.
In a first introduction to the concept of area, students should understand that a rectangle can be made up of a grid of smaller squares. Count the squares that fit within a rectangle to find the total number of squares.	New standard. Fractions were introduced in 2nd grade under the old standards, but not in this format. Area was not introduced.

Grade 3	
New: Common Core Standards	**Old: California State Standards**
Students are required to solve two-step word problems using addition, subtraction, multiplication or division as needed.	Solve problems using two or more operations (addition, subtraction, multiplication or division), but does not specify solving word problems.
Understand that a fraction can be represented on a number line between zero and 1.	New standard. Previously, students were required to add and subtract fractions, but no mention is made of understanding that a fraction is less than 1.
Tell and write time to the nearest minute.	New to 3rd grade. Previously, this was a 2nd grade standard.

Measure and estimate liquid volumes and masses of objects.	Similar to old standard calling for students to estimate and measure the length, liquid volume and mass of an object.
Recognize that shapes in different categories (i.e. squares and rectangles) can share attributes (i.e. both have four sides) and that those shared attributes can define a larger category (i.e. quadrilaterals).	Similar to old standard calling for students to identify the attributes of quadrilaterals (i.e. parallel sides for a parallelogram, equal side lengths for a square), but with emphasis on how to sort shapes instead of on specific rules about shapes.
Multiplication tables are not mentioned in the new standards.	Previously, students were required to memorize the multiplication tables for number 1-10.

Ways to Help Students "Show Their Thoughts" on Paper

Here are six methods, which can help students organize their thoughts. It is recommended that the teacher says to the students: "Show your thoughts" instead of "Show your work". This change encourages students to write more on the paper.

Method 1: Inquiry

Before you teach a new concept, have students fill out the Inquiry Box below. Have the students write or draw any observations. (I see…) What inferences do they have? (I think…) Describe your prior knowledge. (I know…)

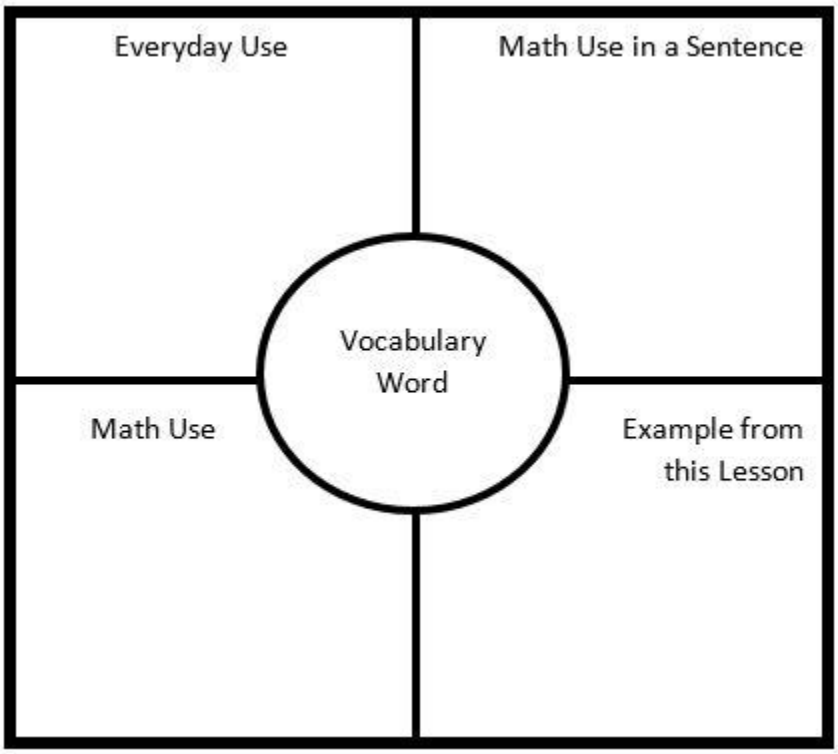
I see… _____

I think… _____

I know… _____

Method 2: Four-Square Chart

Use to review learned vocabulary.

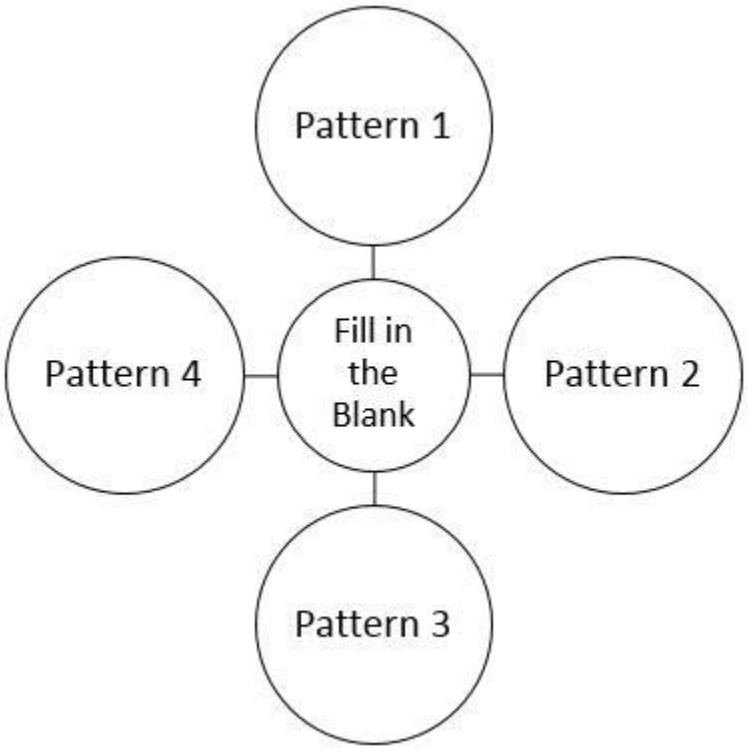


Method 3: Note Taking

Building on the Essential Question:	Today's Notes:
Words I need help with:	
My Math Examples:	

Method 4: Concept Web

Use a Concept Web to solve different types of problems which all have something in common. For example, “Fill in the Blank” can have students look at different patterns and fill in the missing piece of the pattern.



Method 5: Problem-Solving Strategy

Given a word problem, students can fill out this table to help them decode the word problem and figure out how to set up and solve the problem.

Understand	Solve
Plan	Check

Method 6: A Five-Step Formula

Use this method to solve complicated word problems.

5-Step formula

Step 1: What is the Question?

Step 2: What facts are important?

Step 3: What information is not needed?

Step 4: What process $(+, -, \times, or \div)$ should be used?

Step 5: Use the process and the important information to solve the problem.

Classroom Instruction and Discussions

Method 1: Elbow Buddies

Kids get a kick out of this method. “Now talk with your elbow buddy,” is an instruction where they can only talk with the person who sits next to them, touching elbow to elbow. This works well when the students are seated in pairs or groups of four.

Method 2: Think-Pair-Share

This method is very popular, easy, and has been around for a long time – because it works! To start a deep discussion with your students they need to be prepared.

Pose the question to your students. Have them take some predetermined time to **THINK** about their answer. No talking. No writing. No researching. Just thinking. Have them share their thoughts with their Elbow Buddy. Only a **PAIR** of students should be whispering their thoughts back and forth. Then ask the class to raise their hands if they would like to **SHARE** what they thought of or what their Elbow Buddy thought of.

Method 3: Cooperative Learning

Cooperative Learning is very important in Common Core mathematics learning. Students need to share ideas. Having students work on group projects and group presentations will help them be successful in high school and beyond.

Method 4: Direct Instruction

Direct instruction is when the teacher is in the front of the classroom and the students are silently taking notes. There is very little interaction.

One of the founders/authors of the original Common Core idea said that Direct Instruction should be about 20% of total learning. 80% of the time, students should be working in some other fashion. Use direct instruction when the students need to learn a difficult topic.

Method 5: Guided Instruction

Guided Instruction is better than Direct Instruction. With Guided Instruction, the students are interacting with the teacher. The teacher will ask questions to lead the instruction and learning.

Method 6: Inquiry

With Inquiry, students come up with their own questions about what is presented in front of them. Most of the time, this method is used in Science, but it can also be used in mathematics.

Technology Resources for Your Classroom

And it is all FREE!! Or most times, the school will have a subscription, so it is free for you.

Khan Academy

Khan Academy is more than just mathematics. Many subjects are found on this free website. Not only will it be a semester long project for you, but I hope you use this website in your future classroom to help your future students keep up in mathematics.

See Canvas for Math 130's instructions for your semester long project

Google Classroom and Google Forms

In this technology society, it makes sense for students to take exams online. With Google Classroom, a Chromebook and Google Forms, students can take an exam online; where Google locks the Chromebook. Students are not able to visit any other websites while taking the assessment. If they try, Google will cut their assessment score in half and send the teacher an email.

You Tube

It has been said that one can learn anything on YouTube. And it is true. Use YouTube in your classroom to show videos which might help the students "by-in" to what you are trying to teach them.

edPuzzle

This is a wonderful resource. Teachers and students can use to make videos.

Dreambox

This is a fun game based website where the students have to solve grade level appropriate math problems to advance on to the next level and earn gold coins.

ABCya!

Another game based website where the students must solve the math problem to advance. Here the students' games can be assigned by Common Core standards.

Zipgrades

Zipgrades are the new version of scantrons. You use their paper (free download) and your phone to grade their scantron. Works great most of time, to be honest. But if your school does not have a scantron machine, this could be the next best option.

Desmos

Desmos.com is a mathematicians dream come true. It can graph virtually anything. This is a great teaching tool if your classroom has a smart board and you are teaching pre-algebra in 6th grade.

Geometers Sketchpad

Another great tool for teachers. Easily make geometry shapes. Ask your school to buy a license.

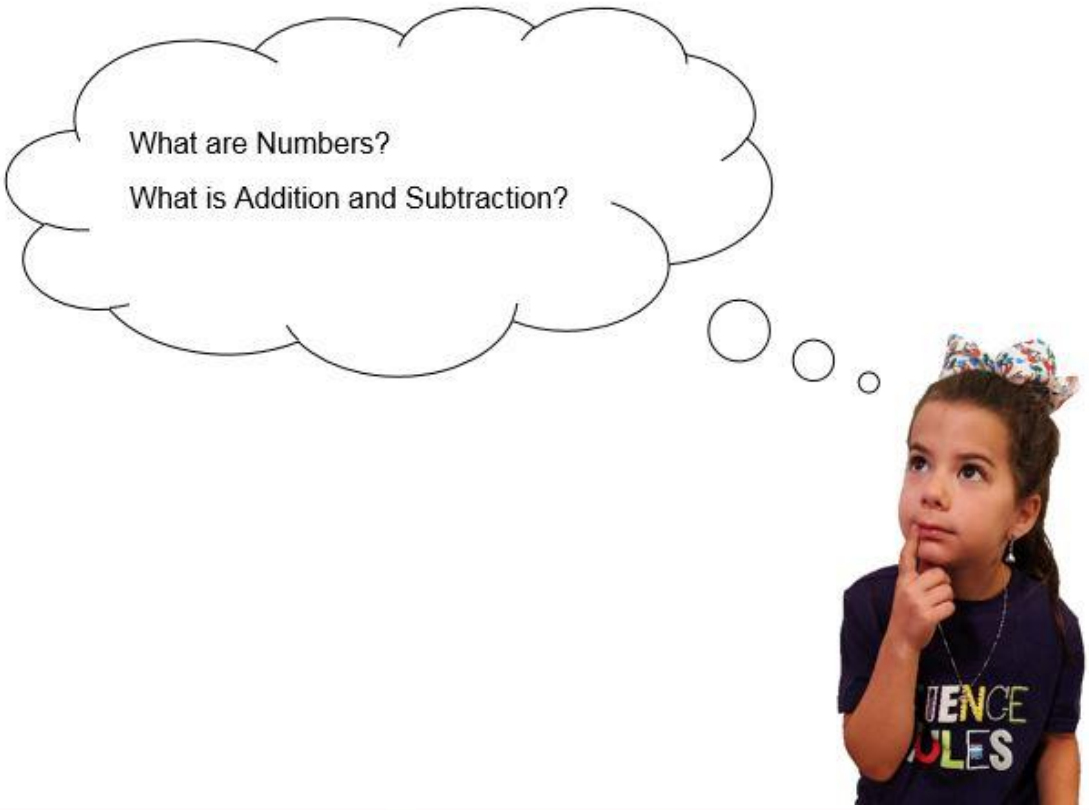
Kuta Software

Kuta makes “drill and kill” worksheets for grades 5 and 6 (it is mostly used for junior high and high school teachers.) Free, already made, worksheets can be found at kutasoftware.com. To make your own worksheets, have your school buy the license.

2 - Empathy and Primary Mathematics

Lesson 2.1 - The Why

The Essential Questions



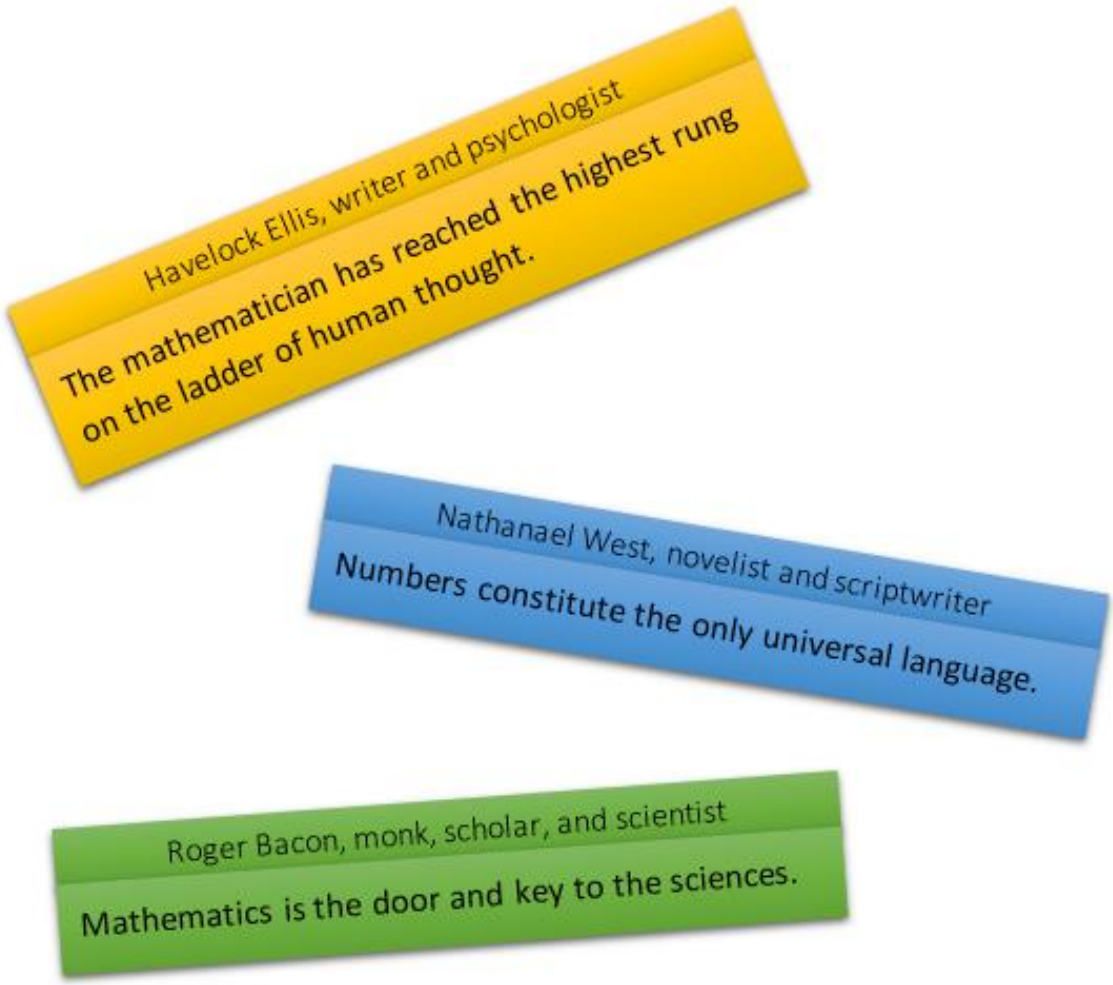
BEFORE YOU BEGIN CHAPTER 2, ANSWER THE ESSENTIAL QUESTIONS HERE:

Why are Teachers Learning this Material?

Empathy.

Do you remember struggling with very basic math? Do you remember using your fingers to add two plus three? Chances are you do not, so this chapter will give you an opportunity to relearn basic math for the first time.

Why are Elementary School Students Learning this Mathematics?



Lesson 2.1 - Practice Problems

1. Why do you want to be a teacher?

2. If you had a choice (and most first year teachers do not), which grade level would you want to teach? Why?

3. Which mathematical topics were the most difficult for you in elementary school? Are they still difficult for you now?

4. Why do you think mathematics is very important subject to learn?

Lesson 2.2 - Units of Measurement

How Common Core Changed Word Problems

Before the mathematicians wrote Common Core, they sat down with Physicists and asked what they would want from Math Teachers: The number one response was UNITS! Students who take physics are notorious for leaving off the units from their answers. Therefore, Common Core made learning units a standard, for grades one through five:

Grade 1: Measure lengths indirectly and by iterating length units.

Grade 2: Measure and estimate lengths in standard units.

Grade 3: Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.

Grade 4: Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft. is 12 times as long as 1 in. Express the length of a 4 ft. snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36),...

Grade 5: Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

Mathematical Practices for Teachers 6: *Attend to Precision (Did I Label My Answer?)*

Example 1

Type	Units
Height (how tall?)	Feet, meters, miles, yards, etc
Weight (how much?)	Pounds, kilograms, ounces, liters, quarts, etc
Speed (how fast?)	Miles per hour (mph) and kilometers per hour (kph) and feet per second (f/s or fps)
Temperature (how hot?)	Celsius C^o and Fahrenheit F^o

Example 2

Quantity versus Quality	
Quantifiable (Measureable units)	**Qualitative (Descriptive Units)**
Feet	Feelings
Gallons	Colors
Miles per Hour	Opinions
Weight	Taste
Money	Sound
Asking about something about a large group of people	Asking about something just about yourself

Lesson 2.2 - Practice Problems

Use Google to help you answer these questions:

1. How tall is the Eiffel Tower?

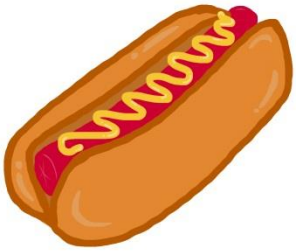
2. What is the average weight of a newborn baby?

3. What is the wealth of America?

Lesson 2.3 - Word Problems

Partner Activity 1

Rosie ate $2\frac{3}{4}$ hot dogs. Edward ate $1\frac{1}{2}$ hot dogs. What part of all the hot dogs together did Rosie eat by herself?



The Trick to Mastering Mathematics

Teach yourself how to do the systematic procedure for the easier version of the problem, which you know very well. Then, take that procedure and apply it to the harder problem.

Rethink the Hotdog problem this way:

Rosie ate five hotdogs. Edward ate three hotdogs. What part of all the hot dogs together did Rosie eat by herself?

Solution:

Step 1: Add the amount of hotdogs. There are eight total hotdogs.

Step 2: Divide Rosie's amount by the total. Rosie ate five of the eight.

Partner Activity 2

Cara is riding in the South Train leaving Los Angeles and heading towards San Diego. Madeline is riding in the North Train leaving San Diego and heading towards Los Angeles. The South Train is traveling 84mph while the North Train is traveling 92mph. The distance between the two cities is 132 miles. Assuming the trains leave at the exact same time, how long will it take them to meet? *(Hints: Draw it out and set up a proportion.)*

Partner Activity 3

One number is four times as large as another number, and their sum is 5285. What are the two numbers? *(Show your solution with squares.)*

Lesson 2.3 - Practice Problems

1. *Grade 1:* Some bikes and trikes are on the playground. There are seven seats and nineteen wheels. How many bikes are there? How many trikes are there? *(Solve without using Algebra.)*

2. *Grade 2:* Pam has three quarters. She wants to buy candy, which costs 54¢. How much change will she have after she buys the candy?

3. *Grade 3:* It takes a rocket 8 seconds to complete one mile. How long will it take the rocket to travel 7 miles?

4. *Grade 4:* Irwin decided to create designs using his cereal. In total, he created nine designs and used 63 pieces of cereal. How many cereal pieces were used in each design? Was there an equal number of pieces in each design?

5. *Grade 5:* Leo has 10 animals at his ranch: turtles, bunnies and chickens. If there were 5 turtles, and 3 bunnies, what fraction of the animals are chickens?

6. *Grade 6:* Ellen wants to build a rectangular fence in her yard, 10 meters by 8 meters. The hardware store owner suggests installing a post every 2 meters. The posts cost $3.42 each. Ellen will also need to buy a glass wall to place in-between each post. The glass costs $50.84 per meter. She also needs to buy one gate for $31.99. How much do all of the materials cost before tax?

Lesson 2.4 - Roman Numerals

Why study Roman Numerals?

Even though Roman Numerals are rare in today's society, they are still used and expected to be understood. They are taught in grades three through five, depending on the district. They can be seen in clocks, the Super Bowl, Film Credits for the copyright date like MCMLXII, preface of textbooks and others like Star Wars Episode VI and WWII.

Basic Table of Roman Numerals:

1	I
5	V
10	X
50	L
100	C
500	D
1000	M

The Complete Table of Roman Numerals:

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
ONES	I	II	III	IV	V	VI	VII	VIII	IX
TENS	X	XX	XXX	XL	L	LX	LXX	LXXX	XC
HUNDREDS	C	CC	CCC	CD	D	DC	DCC	DCCC	CM
THOUSANDS	M	MM	MMM	$\overline{\text{IV}}$	$\overline{\text{V}}$	$\overline{\text{VI}}$	$\overline{\text{VII}}$	$\overline{\text{VIII}}$	$\overline{\text{IX}}$
TEN THOUSANDS	$\overline{X}$	$\overline{XX}$	$\overline{XXX}$	$\overline{XL}$	$\overline{L}$	$\overline{LX}$	$\overline{LXX}$	$\overline{LXXX}$	$\overline{XC}$
HUNDRED THOUSANDS	$\overline{C}$	$\overline{CC}$	$\overline{CCC}$	$\overline{CD}$	$\overline{D}$	$\overline{DC}$	$\overline{DCC}$	$\overline{DCCC}$	$\overline{CM}$

Example 1

For all numbers except 4 and 9, we **ADD** the Roman Numerals together, in order from left to right, greatest value to lowest value.

$$11 = 10 + 1 = XI$$
$$8 = 5 + 1 + 1 + 1 = VIII$$
$$123 = 100 + 10 + 10 + 1 + 1 + 1 = CXXIII$$
$$3816 = 1000 + 1000 + 1000 + 500 + 100 + 100 + 100 + 10 + 5 + 1 = MMMDCCCXVI$$
$$7002 = 5000 + 1000 + 1000 + 1 + 1 = \overline{VII}\,II$$

Example 2

For any number that includes a 4 or a 9, we subtract. When we are looking at a Roman Number expression and we see a Roman character **OUT OF ORDER**, which is the clue to **SUBTRACT**!

$$9 = 10 - 1 = IX$$
$$4 = 5 - 1 = IV$$
$$1400 = 1000 + (500 - 100) = MCD$$
$$29452 = 10000 + 10000 + 10000 - 1000 + (500 - 100) + 50 + 1 + 1 = \overline{IXXX}\,CDLII$$

Partner Activity 1

Convert Between Roman Numerals and Our Current Decimal System:

1. MMXLV = _____
2. MDCCLXXXIX = _____
3. 1993 = _____
4. 5495 = _____

Lesson 2.4 - Practice Problems

Convert Between Roman Numerals and Our Current Decimal System:

1. 84759

2. MMXX

3. MCMLXXXII

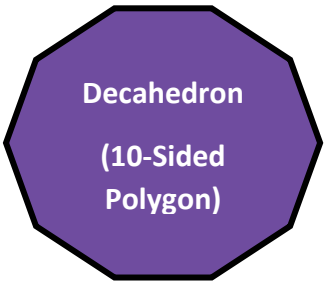
4. 1764

5. 50000

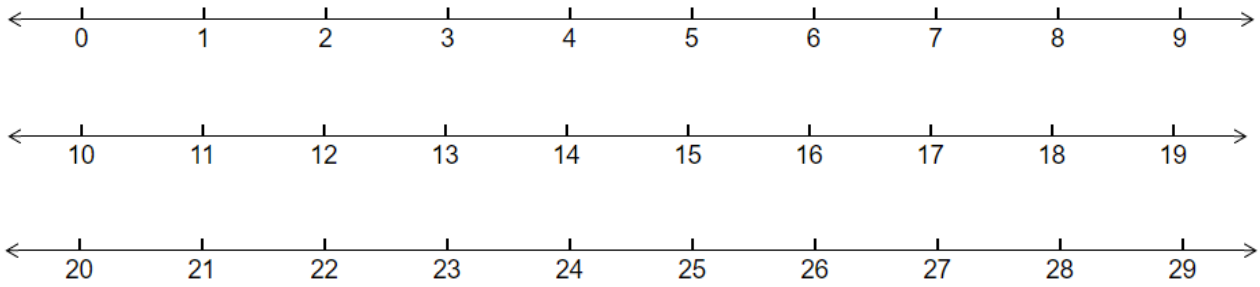
6. $\overline{XX}LXXXIX$

Lesson 2.5 - Different Bases and Their Number Lines

The way we count, the way we visualize numbers, all starts from a number line. As early as Kindergarten, students learn how to read and use a number line. Our current number system is called the Decimal Number System (or Hindu-Arabic). Notice that Decimal begins with DEC like DECahedron (a 10 sided polygon) and we have 10 characters in our numbering system: zero through nine. So, even though we live in a Base 10 system (because we have characters for zero through nine), the number ten is represented by two characters (digits), one and zero.



Below is the number line for Base 10. Notice how it is broken up into rows. We will be using the number lines as rows for the sake of our lesson. However, when you teach the number line to the elementary school students, the number line will be one continuous row (to set up for negative numbers in second grade) OR set up like below only going from 1 to 10 or 0 through 10.



There are an infinite amount of different bases and an infinite amount of corresponding number lines. Below are three different examples, written like we have Base 10 on the previous page.

Base 2 Number Line (reads from left to right, then top to bottom). The fifth number in the number line is 101_{two}.

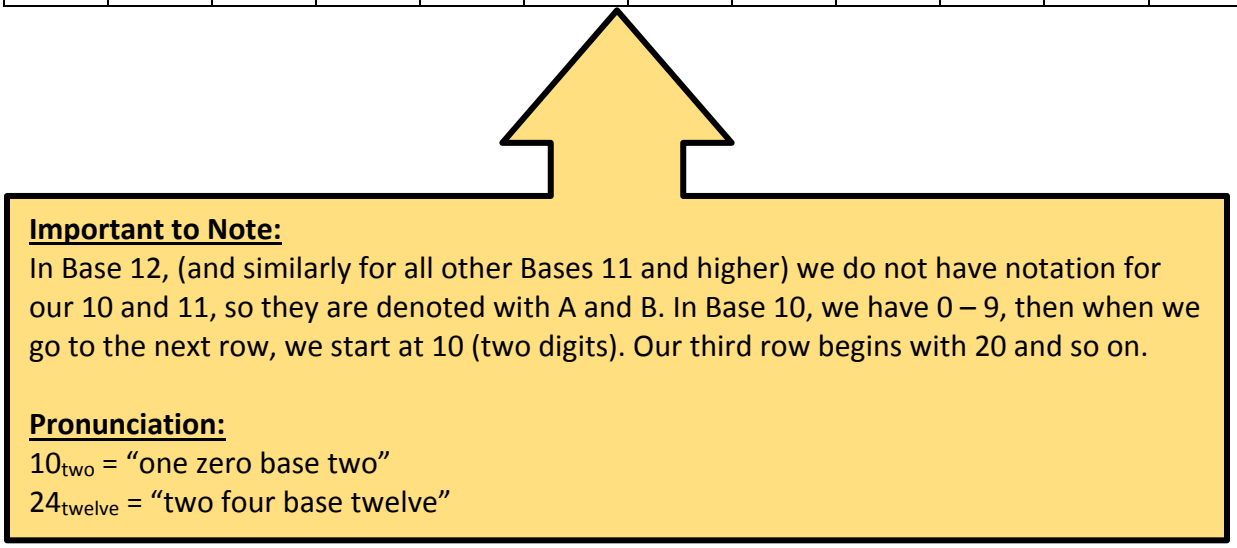
0	1
10	11
100	101
110	111
1000	1001
1010	1011
1100	1101
1110	1111
10000	10001

Base 8 Number Line (reads from left to right, then top to bottom). The 10^{th} number is 12_{eight}.

0	1	2	3	4	5	6	7
10	11	12	13	14	15	16	17
20	21	22	23	24	25	26	27

Base 12 Number Line (reads from left to right, then top to bottom). The 15^{th} number is 13_{twelve}.

0	1	2	3	4	5	6	7	8	9	A	B
10	11	12	13	14	15	16	17	18	19	1A	1B
20	21	22	23	24	25	26	27	28	29	2A	2B



Important to Note:
In Base 12, (and similarly for all other Bases 11 and higher) we do not have notation for our 10 and 11, so they are denoted with A and B. In Base 10, we have 0 – 9, then when we go to the next row, we start at 10 (two digits). Our third row begins with 20 and so on.

Pronunciation:
10_{two} = "one zero base two"
24_{twelve} = "two four base twelve"

Where are different bases used?

Base 2: Computers use Base 2, just zeros and ones for all of their programming. Your phone operates in only zeros (OFF) and ones (ON).

Base 5: This was one of the very first systems of counting, since we have five fingers.

Base 8: Think of Base 8 as the mathematics for the Cartoon Universe. Many Cartoons have only eight fingers. We have 10 fingers and live in a Base 10 universe. Cartoon have eight fingers and live in a Base 8 universe. The Cartoon character does not have a character for eight or nine items, as we do in our universe.

Base 12: Base 12 is rare to find within history. However, we currently have 12 hours on the clock and 12 months in the year. There were a few tribes in Africa and India which used the duodecimal (base 12) system.

Base 20: The Mayans used a Base 20 system, and invented the concept of zero.

Base 60: An extreme example is base 60, which was used by the Babylonians (about 4000 years ago) which is now current day Iraq. They had characters for 1 – 59 items. (The concept of "zero" was not discovered yet.) However, this is where our current concept of 60 minutes and 60 seconds come from.

Lesson 2.5 - Practice Problems

1. What is wrong about 413_{four}?

2. How do you pronounce 542_{six}?

3. Write four rows worth of the number line of Base 3.

4. If you were not familiar with the Base 10 system, which base system do you think would work best for our society and culture?

Lesson 2.6 - Converting Between (our) Base 10 and Any Other Base (and vice versa)

To convert any number in (our base) Base 10 to any other base, we must use basic division with remainders. Do not divide using decimals; the algorithm will not work.

Example 1

Convert from (our) Base 10 to (weird) Base _____

Change 236_{ten} to _____ five

Keep **dividing by 5**, until your quotient is zero.

236 ÷ 5 = 47 r **1**

47 ÷ 5 = 9 r **2**

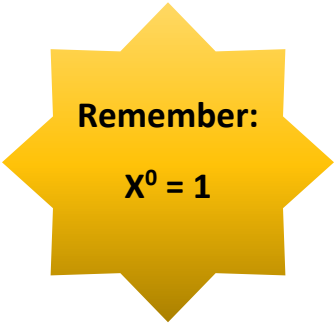
9 ÷ 5 = 1 r **4**

1 ÷ 5 = 0 r **1**

Now write your remainders backwards! **Answer: 1421_{five}**

Example 2

Convert from (weird) Base ____ to (our) Base 10.



First, notice how to break down 602_{ten}: $602 = 6(10^2) + 0(10^1) + 2(10^0)$

Now, use the same approach to change 602_{eight} into Base 10

$$6(8^2) + 0(8^1) + 2(8^0) = 386_{ten}$$

Example 3

Convert 5361_{seven} into Base 10:

$5(7^3)+3(7^2)+6(7^1)+1(7^0)=1905_{ten}$

Partner Activity 1

1. Convert the base 10 numbers into base 4

   A. 30_{ten} = _____ four

   B. 2103_{ten} = _____ four

   C. 16_{ten} = _____ four

2. Convert the base 5 numbers into base 10

   A. 30_{five} = _____ ten

   B. 2103_{five} = _____ ten

   C. 16_{five} = _____ ten

Think carefully about 2C!

****For extra practice, click here.*

Lesson 2.6 - Practice Problems

1. Write the following Base 10 numbers into the new Base.

a. 5567 into Base 9

b. 12 into Base 4

c. 100 into Base 3

d. 73 into Base 2

2. Write the following numbers into Base 10.

a. 64_{seven}

b. 157_{eight}

c. 1001001_{two}

d. 84671_{eleven}

Lesson 2.7 - Place Values with Different Bases

Base 10		Base 2		Base 8		Base 12	
10^0	ones	2^0	ones	8^0	ones	12^0	ones
10^1	tens	2^1	twos	8^1	eights	12^1	twelves
10^2	hundreds	2^2	fours	8^2	sixty-fours	12^2	one-hundred forty-fours
10^3	thousands	2^3	eights	8^3	five-hundred twelves	12^3	one-thousand seven-hundred twenty-eights

Notice that all the words are plural!

Note: $8^3 = 512$ and $12^3 = 1728$

Lesson 2.7 - Practice Problems

In our Base 10 system, the place value for 10,000 is called the ten thousands. Write the equivalent name for the same place value for:

1. Base 3

2. Base 5

3. Base 9

4. Base 14

Lesson 2.8 - Operations in Different Bases

First, let us set up the number lines. We plan to add in base six and then (separate exercise) base three. We are not adding mixed bases in this book, but yes, it is a thing. Google it.

Second, add it vertically. Rewrite the problem. Carry your numbers that are greater than or equal to 10, just like you did in elementary school. But count using the number (actually count it with your finger, swallow your pride and do it).

Base Six Number Line:

0	1	2	3	4	5
10	11	12	13	14	15

Base Three Number Line:

0	1	2
10	11	12

$5415_{SIX} + 3042_{SIX}$

$$\begin{array}{r} 5415_{six} \\ +\ 3042_{six} \\ \hline 12501_{six} \end{array}$$

$1202_{THREE} + 1022_{THREE}$

$$\begin{array}{r} 1202_{three} \\ +\ 1022_{three} \\ \hline 10001_{three} \end{array}$$

Now, we will subtract in different bases. For this example, we will use base seven and eight respectively. Below are the corresponding number lines. Again, use your fingers to count on the number line (everyone else is doing it). Borrow, when appropriate, like you did in elementary school.

Base Seven Number Line:

0	1	2	3	4	5	6
10	11	12	13	14	15	16

Base Eight Number Line:

0	1	2	3	4	5	6	7
10	11	12	13	14	15	16	17

$5631234_{seven} - 2564310_{seven}$

$$\begin{array}{r} 5631234_{seven} \\ -\ 2564310_{seven} \\ \hline 3033624_{seven} \end{array}$$

$3741_{eight} - 1465_{eight}$

$$\begin{array}{r} 3741_{eight} \\ -\ 1465_{eight} \\ \hline 2254_{eight} \end{array}$$

Partner Activity 1

Add or subtract the following numbers:

1. $65_{\text{nine}} + 41_{\text{nine}} =$

2. $123_{\text{four}} + 301_{\text{four}} =$

3. $540_{\text{six}} - 23_{\text{six}} =$

4. $220_{\text{three}} - 122_{\text{three}} =$

5. $1001001_{\text{two}} - 1011_{\text{two}} =$

Lesson 2.8 - Practice Problems

Add or subtract the following expressions.

1. $435_{six} - 31_{six}$

2. $10010_{two} - 111_{two}$

3. $2100_{three} + 21_{three}$

4. $57643_{eight} + 24677_{eight}$

Extension: Methods of Teaching Mathematics

Part 1

In Canvas, start a discussion thread answering the questions below. You must also reply to at two other classmates to earn full credit.

Think about the most effective math teacher you have ever had:

1. What worked?
2. What made you overcome a mathematical obstacle?
3. Which examples can be carried over to YOUR FUTURE classroom?

Part 2

Choose a math standard and topic (to write about later). See Appendix A. Detailed instructions are in Canvas. Submit your assignment via Canvas.

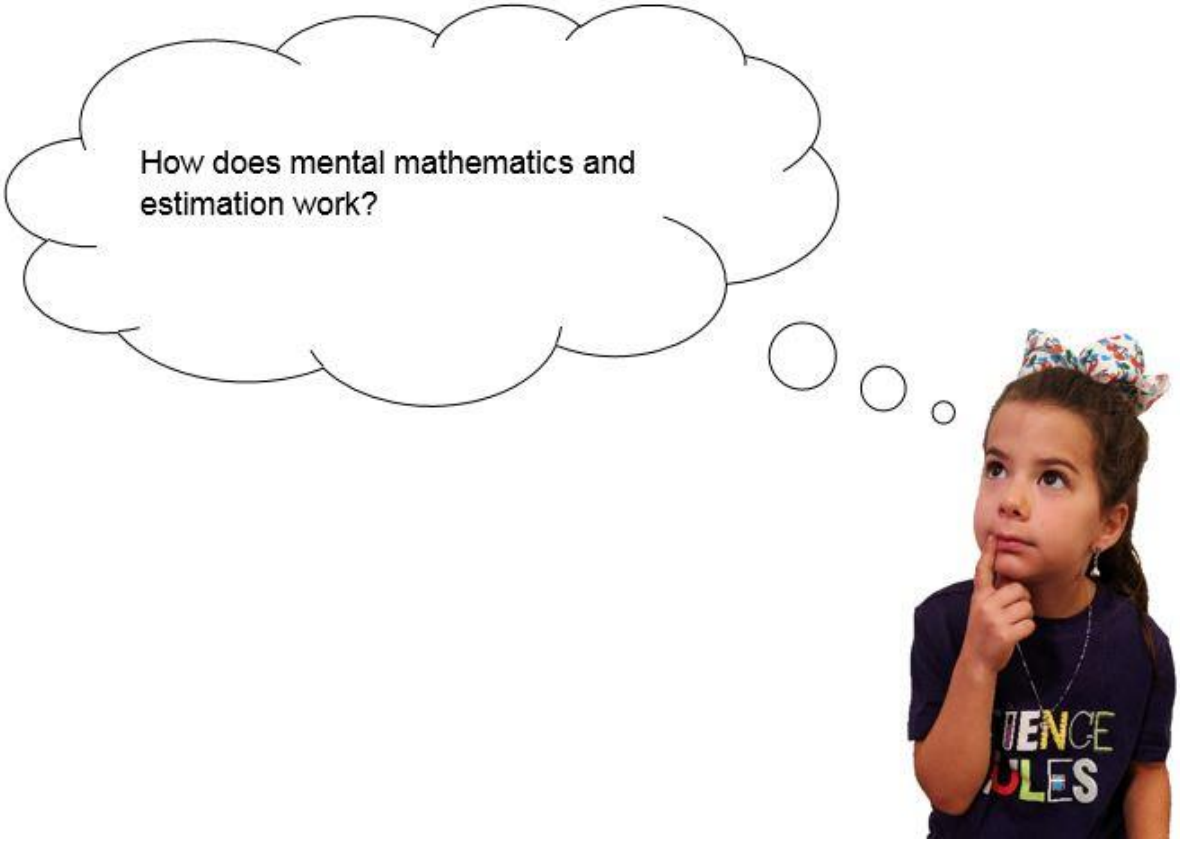
Part 3

Make sure you are working on Khan Academy throughout the semester.

3 - Arithmetic and Mental Mathematics

Lesson 3.1 - The Why

The Essential Question



BEFORE YOU BEGIN CHAPTER 3, ANSWER THE ESSENTIAL QUESTIONS HERE:

Why are Teachers Learning this Material?

Common Core has changed many of the procedures learned throughout this chapter. Most people teach the way they were taught. Most of us did not experience Common Core mathematics in Elementary school, so we need to relearn basic mathematics and retrain our brain on how to teach accurately to the Common Core State Standards. Common Core Mathematics might seem weird or confusing to you, but imagine learning it for the first time, without the prior knowledge. It will make more sense.

Why are Elementary School Students Learning this Mathematics?

One of the major components of Common Core for Elementary School Mathematics is calculating basic mathematics faster and mentally.

Lesson 3.1 - Practice Problems

1. List at least three times in your personal daily life where you use mental math (outside of school).

2. Why do you think it is important for elementary school students to be proficient in mental math and estimation?

Lesson 3.2 - Addition and Subtraction

Example 1- The Three Methods of Subtraction

The Three methods

1. *Missing Addends*
   a. Represent with an algebraic equation
   b. Example: "Mary has seven bananas, but she needs ten. How many more bananas does she need to purchase?"
   c. $7 + x = 10 \rightarrow x = 3$
2. *Take-Away*
   a. Straight forward subtraction
   b. Example: "Jennifer has ten oranges. She sold three of them. How many oranges does she have left?"
   c. $10 - 3 = 7$
3. *Comparison*
   a. Comparing Separate Quantities
   b. Example: "Cory has ten oranges and seven bananas. How many more oranges does Cory have than bananas?"
   c. $10 - 7 = 3$

Algebra in Elementary School

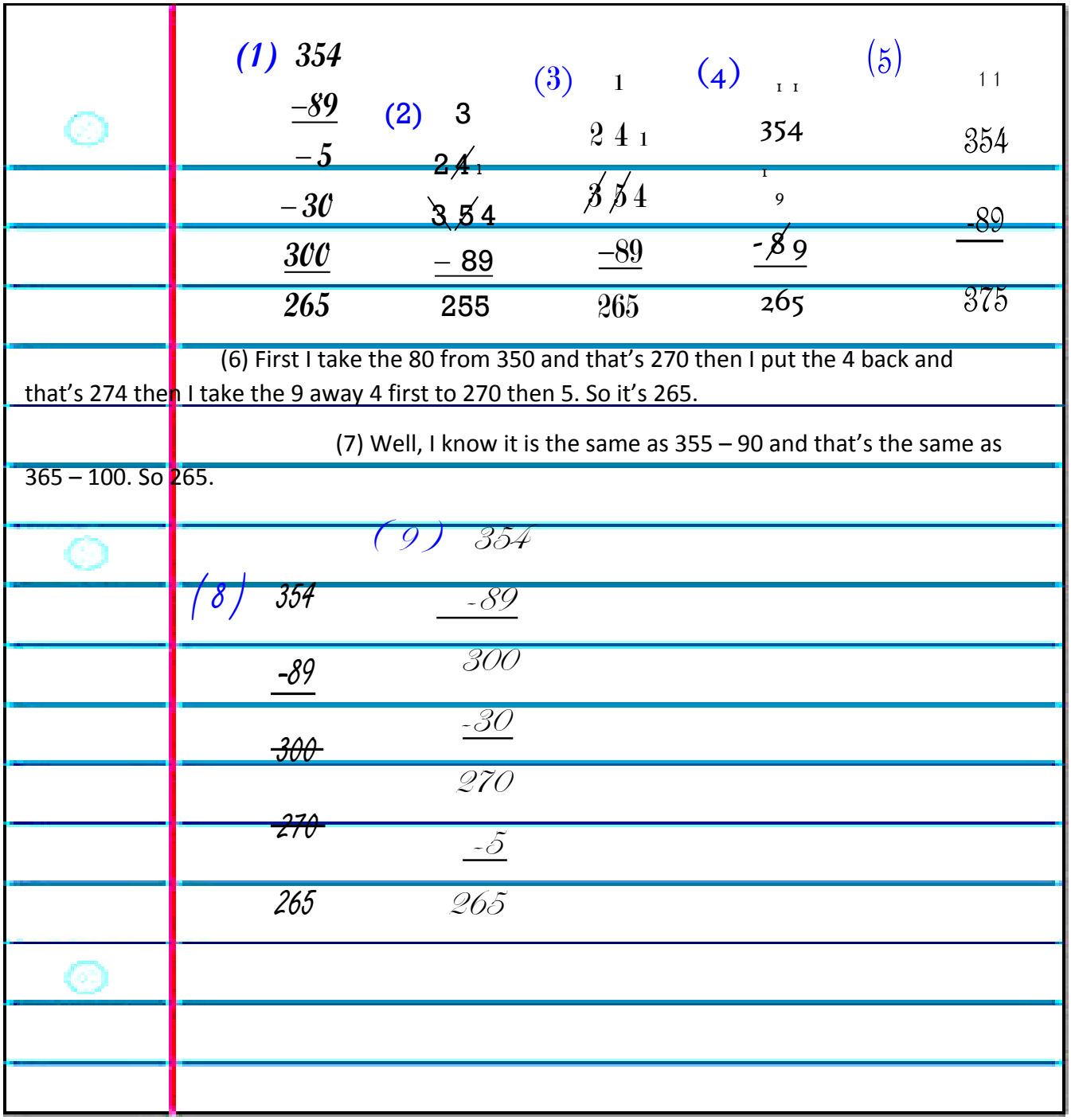
Keep in mind that the introduction of Algebra, (including a variable and solving a one-step equation by subtracting seven on both sides) does not begin until 6th grade. However, students will start thinking algebraically in 1st grade. The same problem can be represented as this: $7 + \square = 10$ *. Students are then asked: "What number goes in the box to make the equation true?"*

Partner Activity 1

By yourself, solve $354 - 89$, any way you like. Then compare and contrast your method to your partner's method. Be prepared to share your method with the class.

Partner Activity 2

Consider the work of nine 2nd graders, all solving $354 - 89$, just like you did a minute ago. Grade each student, as if you were their teacher, using a scale from 1 – 5, 5 being the best. Having the correct answer is only one point out of five. The other four points come from the students' procedure and thoughts. Remember, even if you do not understand HOW they arrived at their correct answer, does not make their procedure incorrect.



Example 2

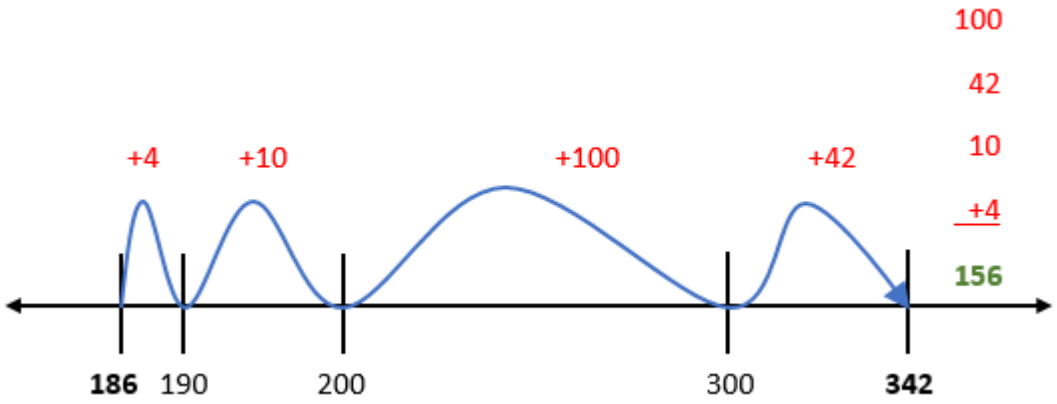
Here is a real-life example of needing to subtract, but actually using addition:

Lance buys some supplies totaling $7.32. He hands the cashier a ten-dollar bill. His change is $2.68. Instead of subtracting 10 – 7.32, the cashier will count UP:

"$7.32 + $1 + $1 + 25¢ + 25¢ + 10¢ + 5¢ + 1¢ + 1¢ + 1¢ = $10.00"

Example 3

Subtract 342 – 186 = 156 using a number line and count UP.



Why are the above examples in the Mental Math section of this textbook? Because doing these problems on paper enough times will train your brain to subtract with mental math and without borrowing.

Partner Activity 3

Subtract the following problems using the methods from either Example 2 or Example 3 above.

1. 753 – 345 = _____

2. 421 – 175 = _____

Lesson 3.2 - Practice Problems

Explain how to solve the following problems using mental math:

1. 56 + 81

2. 1000 – 284

3. 94 + 801

4. 762 – 451

Lesson 3.3 - Multiplication

Example 1

What is the definition of Multiplication? *Repeated Addition*

Common Core has slightly changed the WHY behind Repeated Addition

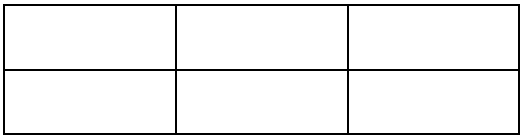
1. Repeated addition
    a. Watermelons cost $3 per pound. How much does a watermelon cost which weighs five pounds?
        i. We could answer this question two different ways
            1. $5+5+5=5\times3=\$15$ (3 groups of 5)
            2. $3+3+3+3+3=3\times5=\$15$ (5 groups of 3)
        ii. Which method is correct? *The first one, only*
        iii. Why? *Two reasons.*
            1. Think farther ahead in math: $5^3=5\times5\times5$, which matches how we write $5\times3=5+5+5$. Do keep in mind that multiplication is communitive, and $5\times3=3\times5$.
            2. The watermelon is the subject of the problem. Its weight is the first issue. We pick up a watermelon and then look at the price. 5 pounds multiplies to $3 because we have 5 pounds for each dollar.
    b. Are we able to use repeated addition for decimals like $0.3\times6=1.8$
        i. Six boxes weigh 0.3 pounds each. What is the total weight?
            1. Are you able to write a repeated 6, 0.3 times? No. So instead we write: $6\times0.3=0.3+0.3+0.3+0.3+0.3+0.3=1.8$
        ii. What is 30% of 6 pounds?
            1. $6\times0.3=0.3+0.3+0.3+0.3+0.3+0.3=1.8$ (6 groups of 0.3)

Example 2 - Sketching Multiplication

Common Core is very big on visualizing mathematics. Students are expected to make drawings to show how multiplication works:

Sketch 3×2

Sketch 2×3



Example 3 - Multiplying with Decimals

Step 1: **Multiply 7 to each digit in the top number from right to left.**

Step 2: **Multiply 4 to each digit in the top number from right to left.**

Step 3: **Multiply 2 to each digit in the top number from right to left.**

Step 4: Add

Step 5: **Move the decimal three spots to the left.** Since there are three digits to the right of the decimals, we move the final decimal three spots to the left. (**bold numbers**)

$$
\begin{array}{r}
123.\mathbf{8} \\
\times 2.\mathbf{47} \\
\hline
28666 \\
49520 \\
+247600 \\
\hline
305.\mathbf{786}
\end{array}
$$

Lesson 3.3 - Practice Problems

1. Expand out 7 X 4

2. Expand out 2 X 6

3. Multiply 4.61 X 7.94 (do not round)

4. Multiply 516.4 X 0.347 (do not round)

Lesson 3.4 - Division

Example 1 - Model Division with Repeat-Subtraction

Leslie has 15 cookies to share with her friends. Each friend wants three cookies. How many friends can Leslie share her cookies with?

$15-3=12-3=9-3=6-3=3-3=0$

There are five groups of cookies, *so Leslie is able to share with five friends.*

This procedure is called Measurement Division. Measurement is any movement across the number line. On the number line, we are starting at 15 and moving down to zero by three's.

Example 2 - Model Division with Sharing

Charlie has 15 balloons and three children. How many balloons does each child receive?

Each child receives five balloons.

This procedure is called Sharing Equally OR Partitive Division. Think of it like using the distributive property. Charlie is equally distributing a balloon to each child. Once each child has one balloon, Charlie then equally distributes another round of balloons to each child. Charlie stops when he runs out of balloons.

Partner Activity 1 - Dividing by Zero

What is $5 \div 0$ also written as $\frac{5}{0}$? Why?

What is $0 \div 5$ also written as $\frac{0}{5}$? Why?

What is $0 \div 0$ also written as $\frac{0}{0}$? Why?

**How would you explain dividing by zero to a 4th grader??

Lesson 3.4 - Practice Problems

1. Sophie needs to pass out papers to her coworkers. She has 28 papers and each coworker needs four papers. How many coworkers does she have? What type of division is this?

2. Later that day, Sophie speaks to ten other coworkers who need different papers. She runs off 30 copies. How many papers did each coworker receive? What type of division is this?

Lesson 3.5 - Estimation and Rounding

What is the difference between estimation and rounding? First, let us look at the definitions from Google:

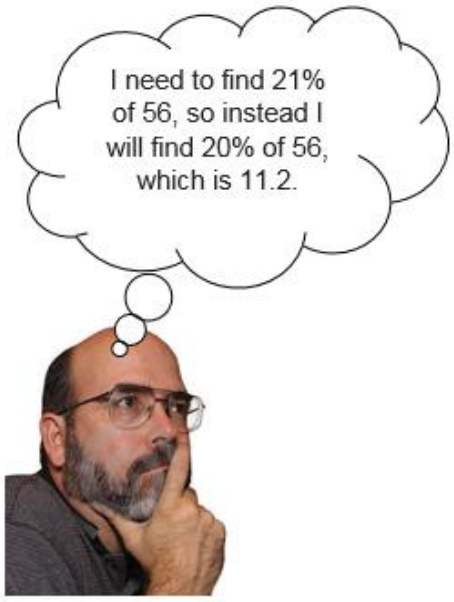
Estimation:

A rough calculation of the value, number, quantity, or extent of something.

Rounding:

Alter (a number) to one less exact but more convenient for calculations.

Now let us look at some examples and determine which one is which:



Irwin's example is **Estimation**. He is estimating an answer by using easier numbers.

Mary's example is **Rounding**. She is rounding up 0.76 to 1, to make the numbers easier for her.

One of the main components of Common Core was to make sure that students could perform mental math exercises with ease. Therefore, there is a strong emphasis on percentages, estimation, rounding and multiplication.

Mental Math Percentages

(use these to help you with Example 1)

10%: Move the decimal once to the left

5%: Find 10% and divide by 2

20%: Find 10% and multiply by 2

1%: Move the decimal twice to the left

Example 1

Restaurant Split

For dinner, on Friday night, Chris and Maddy dine at Seven Seas Seafood. They decide to leave their phones in the car, so that there are no distractions and actually have some real conversation! The dinner went very well and the bill comes. Maddy decided to be nice and pay the entire bill; after all, she did order the crab buffet. Now, her phone is in the car and she needs to calculate tip and total. Chris says, "I think you should tip 17%. The service was good, but not great. And do it quick, I want to go home." Luckily, Maddy remembered how to use her amazing estimation skills and tipped $20.

$$17\% = 10\% + 5\% + 1\% + 1\%$$

$$10\% = \$11.658 \approx \$12$$

$$5\% \approx \$12 \div 2 = \$6$$

$$1\% = \$1.1658 \approx \$1$$

$$\$12 + \$6 + \$1 + \$1 = \$20$$

Partner Activity 1

Calculate tip for the following restaurant bills using the method presented in Example 1. Make sure you round to the nearest penny.

1. $85.12

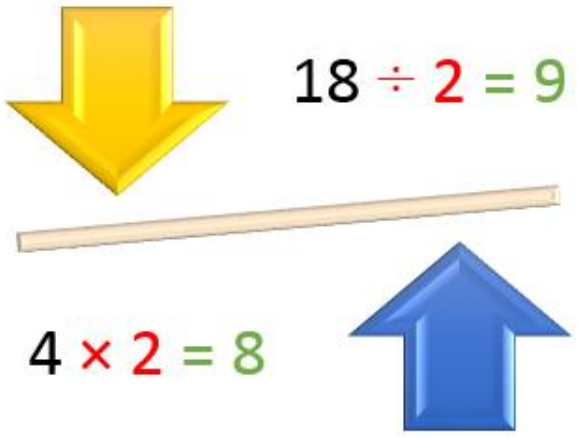
2. $21.47

3. $96.01

Example 2 - Mental Math Multiplication

Growing up, most of us learned the multiplication tables from zero to ten. Maybe, even some of you reached 11 and 12. But what about the higher numbers? Here is Common Core's answer:

We need to multiply **18 × 4**. We can change the expression as long as we keep it balanced. Dividing 18 by 2 AND multiplying 4 by 2 will keep the expression balanced.



Therefore, 18 × 4 is the same as 9 X 8 = 72.

We do not have to use just two. Any number will work! As long as you keep it balanced!

Partner Activity 2

Here are some advanced multiplication expressions. By yourself, write an easier way to evaluate the expression, so you can use mental math. Then solve. Afterwards, compare your methods to your partner's method.

1. 24×3

2. 5×36

3. 16×4

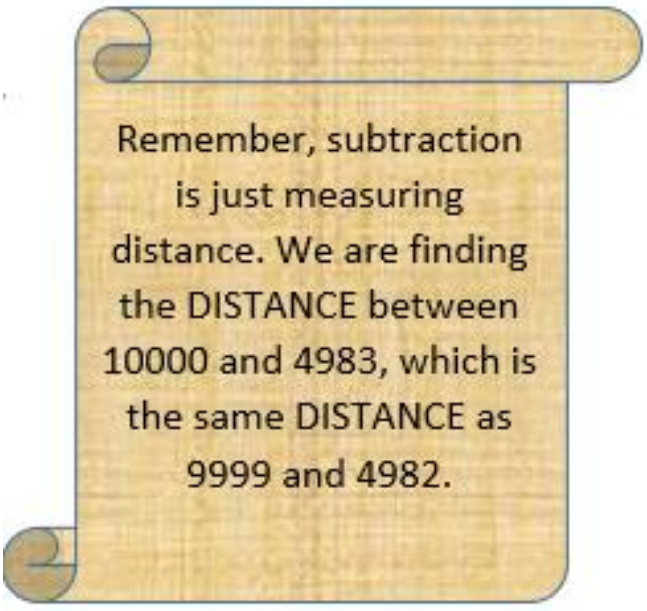
Example 3 - Mental Math Subtraction Trick

One of the hardest things for students to learn is subtracting when the first number (called the minuend) has lots of zeros, like 10,000. There can be confusion with borrowing: which zeros end up being tens and which zeros end up being nines? Common Core has an answer for that, too!

Let's try subtracting $1000 - 4983$ the traditional way, with borrowing:

$$\begin{array}{r} 9\ 9\ 9 \\ 0\cancel{1}0\ \cancel{1}0\ \cancel{1}010 \\ \cancel{1}\ \cancel{0}\ \cancel{0}\ \cancel{0}\ \cancel{0} \\ -\ 4\ 9\ 8\ 3 \\ \hline 5\ 0\ 1\ 7 \end{array}$$

Some students might see a problem like this and be intimidated by the amount of borrowing. An easier way to subtract: First subtract 1 from both numbers.

Going the Distance	
1000 → 9999 and 4983 → 4982 $\begin{array}{r} 9999 \\ -4982 \\ \hline 5017 \end{array}$ ***No. More. Borrowing. EVER.*** 	

Partner Activity 3

Solve these subtraction problems WITHOUT borrowing! Think about what you should **add or subtract to both numbers.**

1. 1000 – 679

2. 513 – 284

3. 16452 – 999

Mental Math Addition and Subtraction in Schools Today

Common Core changed the way students add and subtract. Some schools still using carrying (addition) and borrowing (subtraction), like traditional mathematics. However, some schools encourage to teach the Common Core methods explained above, which will lead to mental math abilities, after enough practice.

Lesson 3.5 - Practice Problems

1. Estimation

a. What is 11% of 150?

b. What is 19% of 400?

2. Rounding

a. Round 16.456 to the nearest hundredth.

b. Round $49.347 to the nearest penny.

c. Round 82.42 to the nearest whole number.

3. Tip

a. The restaurant bill is $43.18. What is a 15% tip?

b. The restaurant bill is $56.79. What is a 20% tip?

c. The restaurant bill is $96.42. What is a 17% tip?

4. Multiply big numbers

a. Use mental math to multiply 4 X 18.

b. Use mental math to multiply 24 X 3.

c. What did you notice about A and B?

5. Subtract from 10000

a. Use the trick in Example 3 to subtract 10000 – 9134.

b. Use the trick in Example 3 to subtract 100000 – 34678.

Lesson 3.6 - Personal Referents

Think, for yourself, personally, how big areas or distances can be for you. It can be anything, as long as it makes sense to you! For example, the distance from Noel's house to the end of the block is ¼ of a mile. So when anybody says, "Let's take a walk, it is just half a mile", Noel knows it is the same distance as from his house to the end of the block and back. Another example, is where Janice knows that 300 people can fit comfortably in the gym. So, if someone says, I think I am going to invite about 300 people to my wedding, she can picture the gym filled up to capacity and have an idea about how large the wedding will be.

Partner Activity 1

Think about personal referents for yourself and then share with your partner:

1. 200 miles

2. 10 feet

3. 50 pounds

Lesson 3.6 - Practice Problems

Write a personal referent for 1 meter, 1 inch, 1 foot, and 1 yard.

Lesson 3.7 - Calculating Percentages

(Mental Math verses pencil to paper)

30 is 20% of what number?

Mental math version: We know that a whole amount is 100%, so five 20%s are 100%. Therefore, we can multiply 30 by 5 and get 150, our answer.

But sometimes, it is not that easy. We need a formula:

$$\frac{is}{of} = \frac{\%}{100}$$

30 is 23% of what number? Use the proportion below, cross multiply, and solve for x. (Algebra)

$$\frac{30}{x} = \frac{23}{100}$$

Partner Activity 1

1. Find 16% of 54.

2. What percent of 97 is 43?

3. 21% of what number is 842.

Lesson 3.7 - Practice Problems

1. Find 94% of 329.

2. What percent of 14 is 13? (round to the nearest hundredth)

3. 64% of what number is 73. (round to the nearest hundredth)

Lesson 3.8 - Extension: Methods of Teaching Mathematics

Part 1

What do you need to be a good (OR GREAT) mathematics teacher!

1. Know your subject matter.

2. Have patience with your students.

3. Be organized.

4. Have good classroom management skills

Part 2

In class, we will be watching videos of good and bad teaching practices. See Canvas for worksheet.

Part 3

Your homework assignment is to observe a teacher (either in person or on youtube.com) and write about their Classroom Management approach (good or bad). Turn this in via Canvas.

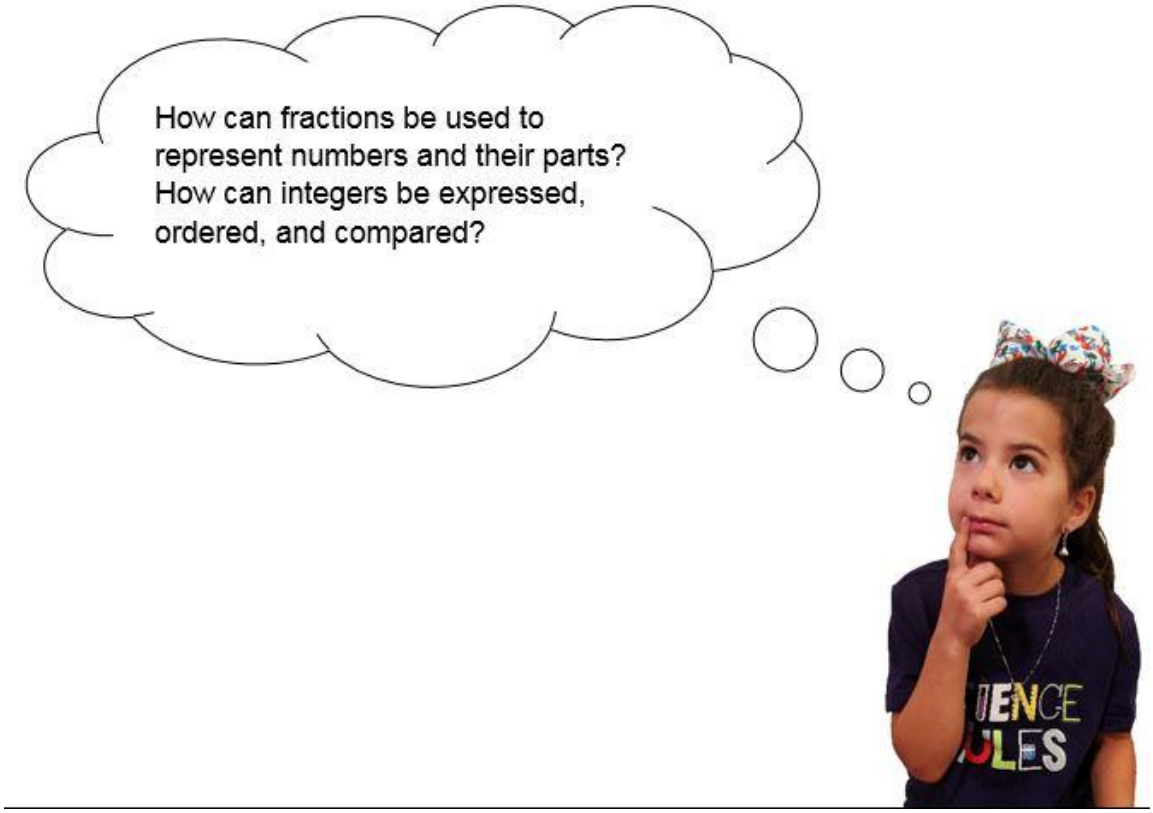
Part 4

Keep working on Khan Academy!

4 - Fractions and Integers

Lesson 4.1 - The Why

The Essential Questions



BEFORE YOU BEGIN CHAPTER 4,
ANSWER THE ESSENTIAL QUESTIONS HERE:

Why are Elementary School Teachers Learning this Mathematics?

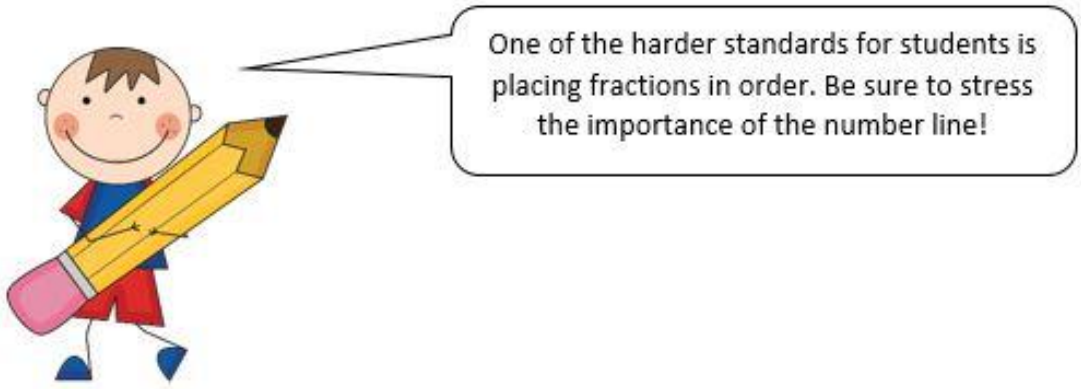
In a Wall Street Journal article from 2013, it states that understanding fractions in third, fourth and fifth grade is a predictor of long-term math achievement. Fractions are the foundation for advanced mathematics. Common Core mandates that fractions be mastered by the end of fifth grade.

Why are Elementary School Students Learning this Mathematics?

Fractions are a necessary component for students. Fractions are used outside of the classroom and career in areas like baking, sewing, construction and some sports. Studies have shown that students who do not master fractions end up having anxiety in their secondary and post-secondary mathematics classes; and end up not preforming as well as other students who did master fractions in elementary school.

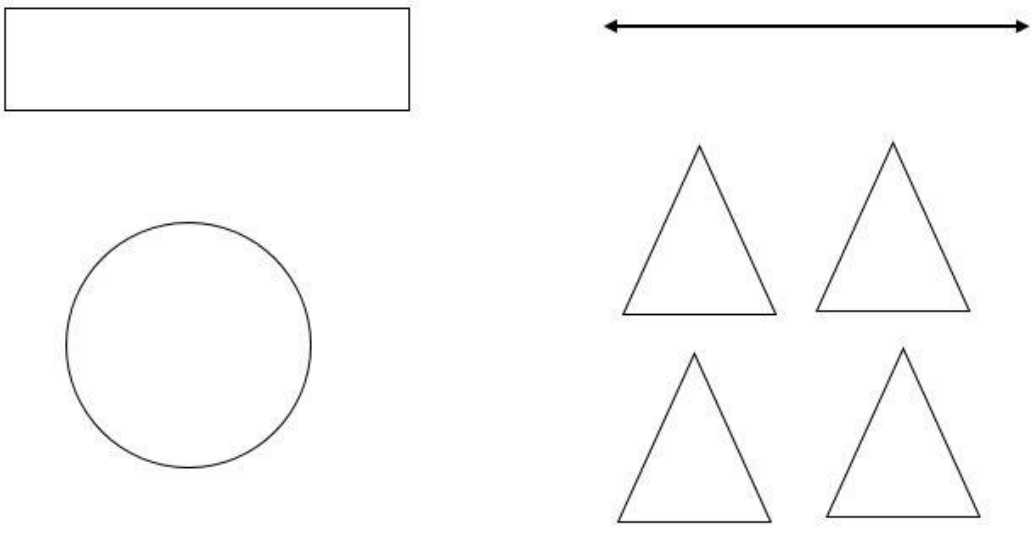
Lesson 4.2 - What are Fractions?

$\frac{\text{Part}}{\text{Whole}} \rightarrow \frac{\text{Numerator}}{\text{Denominator}}$	$\text{Denominator}\sqrt{\text{Numerator}}$	**Fractions are just Division!**



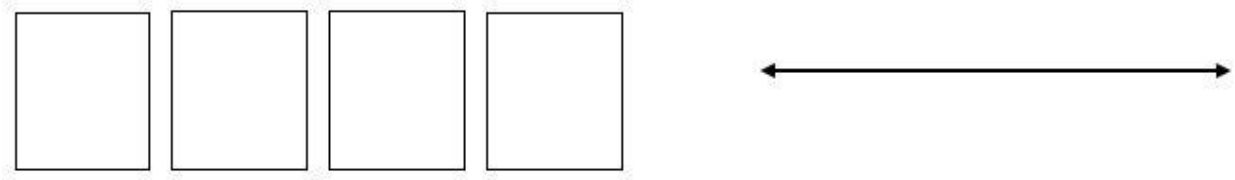
Example 1

Explain the meaning of $\frac{1}{4}$



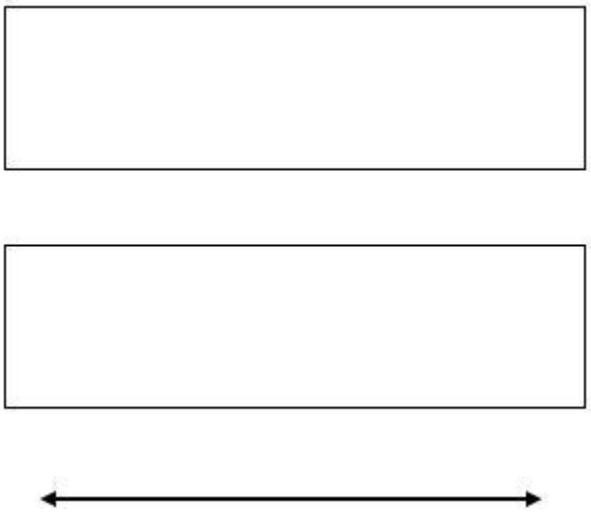
Example 2

Show $\mathbf{3\frac{5}{8}}$



Example 3

Which is bigger? $\mathbf{\frac{1}{3}}$ $\boldsymbol{OR}$ $\mathbf{\frac{1}{4}}$



Example 4

Which is bigger? $\frac{4}{9}\ OR\ \frac{5}{8}$

Use the LCM to make the common denominator. The LCM (9, 8) = 72.

$$\frac{4}{9}=\frac{4\times8}{9\times8}=\frac{32}{72}$$

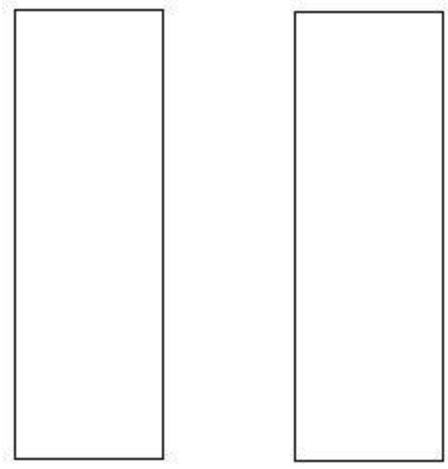
$$\frac{5}{8}=\frac{5\times9}{8\times9}=\frac{45}{72}$$

Since $\frac{45}{72}>\frac{32}{72}$, then $\frac{5}{8}>\frac{4}{9}$

Example 5

Equivalent Fractions

Show that $\frac{1}{2}=\frac{4}{8}$



Example 6

Create Equivalent Fractions

$$\frac{2}{3}=\frac{2\times 4}{3\times 4}=\frac{8}{12}$$

OR

$$\frac{2}{3}=\frac{2\times 11}{3\times 11}=\frac{22}{33}$$

Example 7

A 5th grade class has 11 girls and 13 boys. What fraction of the class has boys?

Solution:

1. Find the total (whole): 11 + 13 = 24 students

2. Write your fraction: $\frac{boys}{class}=\frac{13}{24}$

Partner Activity 1

Describe ways of telling when a fraction is close to…

1. zero
2. One
3. one-half
4. one-third

Partner Activity 2

Organize the fractions by which it is closest to zero, one-half or one.

$$\frac{3}{8}\quad\frac{5}{4}\quad\frac{2}{9}\quad\frac{4}{7}\quad\frac{1}{3}$$

Lesson 4.2 - Practice Problems

1. Explain the meaning of $3\frac{1}{2}$ two different ways.

2. Put the following fractions in order from least to greatest: $\frac{4}{7}, \frac{2}{5}, \frac{1}{9}, \frac{12}{13}, \frac{3}{8}$

Lesson 4.3 - Add, Subtract, Multiply and Divide Fractions

Partner Activity 1

Which expression would you rather add?

$\frac{51}{684}+\frac{43}{684}+\frac{738}{684}$ OR $\frac{1}{8}+\frac{4}{5}+\frac{1}{9}$

Explain to a 3rd grader why:

_____
_____
_____
_____
_____
_____

Now, we will explore why fractions behave the way they do for adding, subtracting, multiplying and dividing:

Example 1

Add Fractions with a Drawing, Number Line, then with Common Denominators

Add $\frac{1}{2}+\frac{1}{3}$ *Why 6 boxes?*

Example 2

Subtract Fractions with a Drawing, Number Line, then with Common Denominators

Subtract $\frac{1}{2} + \frac{1}{3}$ *Why 6 boxes?*

Example 3

Multiply Fractions with a Drawing, Number Line, then with Common Denominators

Add $\frac{1}{2} \; x \; \frac{1}{3}$ *Why 6 boxes?*

Example 4

Divide Fractions with a Drawing, Number Line, then with Common Denominators

Divide $\frac{1}{2} \div \frac{1}{3}$ "Think portions, when it comes to division! *Why 6 boxes?*

Example 5

Why does "multiply and flip the second fraction" work when dividing fractions?

We know that: $\frac{a}{b} \times \frac{b}{a} = \frac{ab}{ab} = 1$

$$\frac{3}{4} \div \frac{2}{7} =$$

$$= \left(\frac{3}{4} \times \frac{7}{2}\right) \div \left(\frac{2}{7} \times \frac{7}{2}\right)$$

$$= \left(\frac{3}{4} \times \frac{7}{2}\right) \div 1$$

$$= \frac{3}{4} \times \frac{7}{2} = \frac{21}{8}$$

Partner Activity 2

Which Operation is Correct?

A stretch of highway is $3\frac{1}{2}$ miles long. Each day, $\frac{2}{3}$ of a mile is repaved. How many days are needed to repave the entire section? **How would you explain to a 5th grader which operation is correct?**

Do we add?

$3\frac{1}{2}+\frac{2}{3}=\frac{7}{2}+\frac{2}{3}=\frac{21}{6}+\frac{4}{6}=\frac{25}{6}=4\frac{1}{6}$ days

Do we subtract?

$3\frac{1}{2}-\frac{2}{3}=\frac{7}{2}-\frac{2}{3}=\frac{21}{6}-\frac{4}{6}=\frac{17}{6}=2\frac{5}{6}$ days

Do we multiply?

$3\frac{1}{2}\times\frac{2}{3}=\frac{7}{2}\times\frac{2}{3}=\frac{14}{6}=2\frac{2}{6}=2\frac{1}{3}$ days

Do we divide?

$3\frac{1}{2}\div\frac{2}{3}=\frac{7}{2}\times\frac{3}{2}=\frac{21}{4}=5\frac{1}{4}$ days

Lesson 4.3 Practice Problems

Add, subtract, multiply or divide the expressions. Use any method.

1. $\frac{3}{4} + \frac{8}{7}$

2. $\frac{8}{7} - \frac{3}{4}$

3. $\frac{3}{4} \; x \; \frac{8}{7}$

4. $\frac{3}{4} \div \frac{8}{7}$

5. $5\frac{2}{5} \div 3\frac{1}{6}$

6. $5\frac{2}{5} - 3\frac{1}{6}$

7. $5\frac{2}{5} \; x \; 3\frac{1}{6}$

8. $5\frac{2}{5} \div 3\frac{1}{6}$

Lesson 4.4 - Models and Operations with Integers

Example 1 - Groups of Numbers

The largest group is the Real Number System. The Real Numbers hold true for all numbers learned in elementary school. The only numbers, which do not belong in the Real Number group, are imaginary numbers, which are not learned until high school.

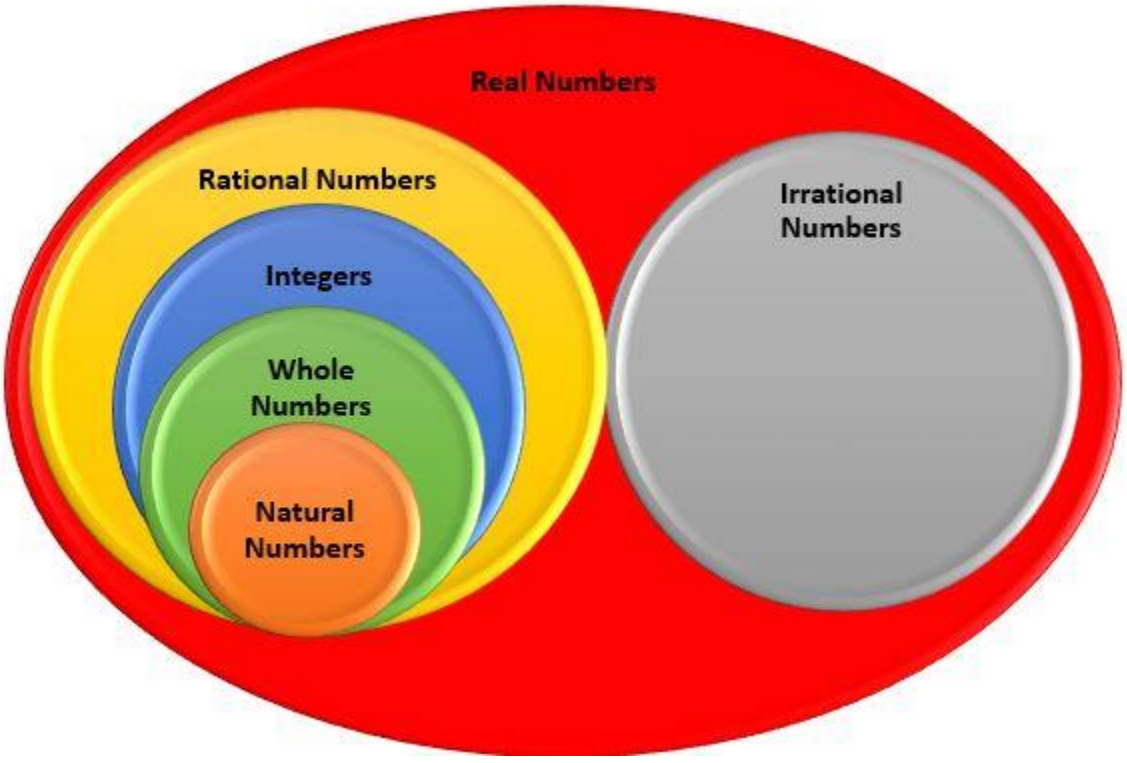
Within the Real Numbers are the Rational and Irrational Groups. Rational numbers are any number which can be written as a fraction, whereas any irrational number is non-terminating (never stops) and non-repeating. If a decimal has a pattern, it can be made into a fraction.

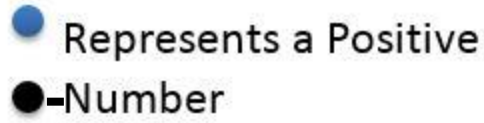
Integers, the next smallest group, include negative numbers, zero and positive numbers, without any fractions or decimals.

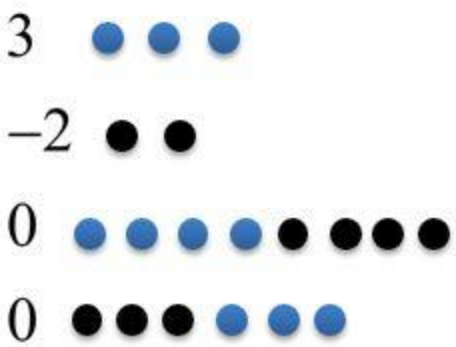
Whole numbers, like Integers, cannot have fractions or decimals. In addition, Whole Numbers start with zero include positive numbers. No negatives.

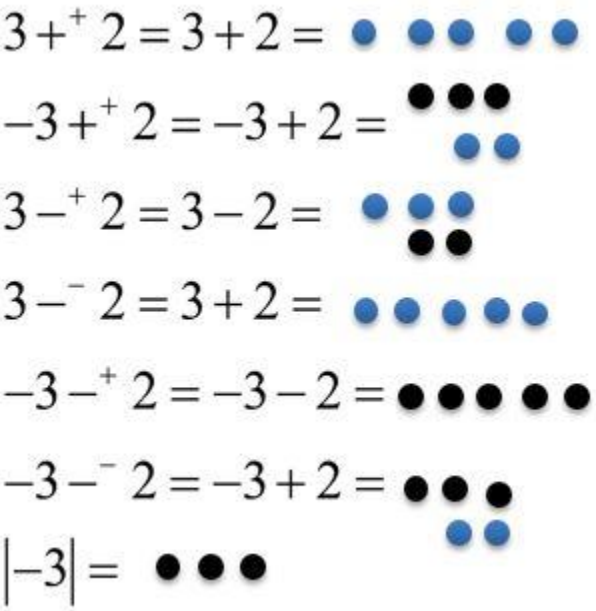
The smallest group of numbers are the Natural Numbers. They are like Whole Numbers, except without zero.

Example 2 - Modeling Signed Numbers



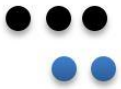

Represents a Positive
Number

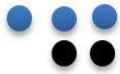

3
−2
0
0


3 + 2 = 3 + 2 =
−3 + 2 = −3 + 2 =
3 − 2 = 3 − 2 =
3 − 2 = 3 + 2 =
−3 − 2 = −3 − 2 =
−3 − 2 = −3 + 2 =
|−3| =

Example 3 - Operations with Signed Numbers

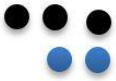
$3 +^{+} 2 = 3 + 2 =$ 

$-3 +^{+} 2 = -3 + 2 =$ 

$3 -^{+} 2 = 3 - 2 =$ 

$3 -^{-} 2 = 3 + 2 =$ 

$-3 -^{+} 2 = -3 - 2 =$ 

$-3 -^{-} 2 = -3 + 2 =$ 

$|-3| =$ 

Example 4 - Operations on the Number Line

$6 + 5 =$ 

$-6 + 5 =$ 

$6 +^{-} 5 = 6 - 5 =$ 

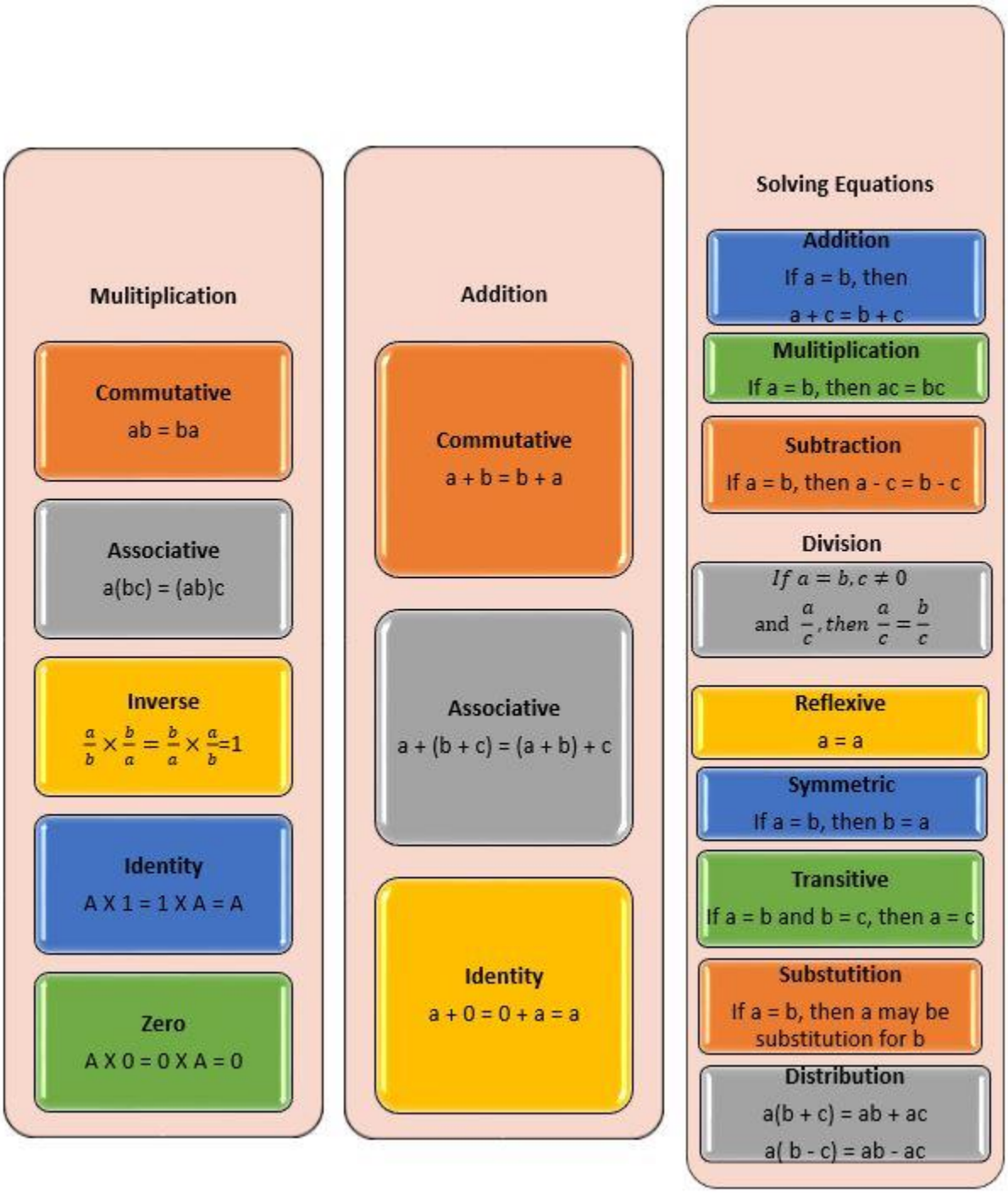
$-3 - 4 =$ 

$3 + 4 =$ 

$6 -^{-} 5 = 6 + 5 =$ 

$|-3| - |-4| = 3 - 4 =$ 

Example 5 - Table of Numerical and Algebraic Properties



Lesson 4.4 Practice Problems

Put the following numbers into their correct group(s).

1. 16

2. 3.14

3. π

4. $\sqrt{16}$

5. $\sqrt{23}$

6. -8

7. 0

8. 1

Add or subtract the following integers.

9. -16 + 45

10. 91 – (-56)

11. 64 – 341

12. 78 + (-89)

13. 232 + 44

State the property used for each equation.

14. 5(4 + 2) = 20 + 10

15. 4(3)(12) = 4(12)(3)

16. 42 = 42

17. If x = 3 and x = y, then y = 3

Extension: Methods of Teaching Mathematics

Part 1

Watch a full 45-minute lesson and write the corresponding lesson plan for it. See Canvas for more detailed instructions.

Part 2

Make sure you are working on Khan Academy throughout the semester.

5 - Number Theory

Lesson 5.1 - The Why

The Essential Questions

How do fractions and integers use the Greatest Common Factor and the Least Common Multiple?

BEFORE YOU BEGIN CHAPTER 5,
ANSWER THE ESSENTIAL QUESTIONS HERE:

Why are Teachers Learning this Material?

By the time many students reach college, applying the rules of Greatest Common Factor (GCF) and Least Common Multiple (LCM) are so ingrained in their minds, that they do not even realize they are using the rules anymore. They then began to forget the rules and just apply the steps needed to get the correct answer. For example, many times college students forget that getting like denominators for adding fractions does involve the LCM OR factoring out a number of a polynomial involves the GCF. This chapter is to remind them where their knowledge came from.

Why are Elementary School Students Learning this Mathematics?

The Greatest Common Factor (GCF) and Least Common Multiple (LCM) are important skills for elementary school students. The GCF allows students to reduce fractions. Mastering the GCF will help students later on with the distributive property (factoring) in Algebra 1 as well as word problems. Learning the LCM in elementary school allows students to add and subtract fractions with unlike denominators.

Lesson 5.2 - Number Theory

Example 1

Factors and Multiples Definition: Let $mn = p$, then m and n are factors of p and p is a multiple of m and n.

- Factors of 12: 1, 2, 3, 4, 6, & 12
- Multiples of 12: 12, 24, 36, 48, 60, …

Hints about Factors and Multiples:

Factors are always **smaller** than the given number, whereas **multiples** are always **bigger** than the given number.

Partner Activity 1

List all the factors and the first four multiples of 30.

Partner Activity 2 - Finding Primes

1. Below are the numbers from 0 to 99.
2. Cross out 0 and 1 (neither prime nor composite) and circle 2 (the first prime)
3. Cross out all multiples of 2.
4. Circle 3 (prime) and cross out all multiples of 3.
5. Circle 5 (prime) and cross out all multiples of 5.
6. Continue this exercise until each number is either crossed out or circled.
7. Write all your circled primes below.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19

20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39

40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59

60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79

80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99

Primes and Composites

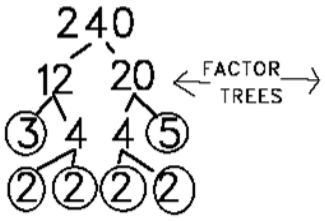
Definitions:

- **Prime number:** any natural number, which has no factors other than 1 and itself
- **Composite number:** any natural number, which has a factor other than 1 and itself
- **Primes**: 2, 3, 5, 7, 11, 13, 17, 19, 23, …
- **Composites**: 4, 6, 8, 9, 10, 12, 14, 15, …
- **Relatively Prime:** two or more numbers with no factors in common, i.e., 7 and 8 *or* 15 and 4

Partner Activity 3

Categorize the following as Prime, Composite or Neither: 0, 1, 2, and any negative number

Example 2 - Prime Factorization (Factor Tree)



FACTOR TREES

$240 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$

$240 = 2^4 \cdot 3 \cdot 5$

$28 = 2 \cdot 2 \cdot 7$

$28 = 2^2 \cdot 7$

Partner Activity 4

Write the prime factorization:

1. 85
2. 350
3. 60

Lesson 5.2 - Practice Problems

List all the factors.

1. 56

2. 145

3. 32

List the first four multiplies.

4. 50

5. 23

6. 8

List the prime numbers.

7. Between 20 and 40

8. Between 60 and 80

9. Between 120 and 150

List the composite numbers.

10. Between 20 and 40

11. Between 60 and 80

12. Between 120 and 150

Write the prime factorization.

13. 540

14. 60

15. 125

Lesson 5.3 - Divisibility Rules

Divisibility Rules	
Divisible by ___?	**The Trick!**
2	Last digit is even
3	Add up the digits and if the sum is divisible by 3 then so is the original number
4	Divide last 2 digits by 4
5	Ends in 0 or 5
6	Rules for 2 and 3 work
8	Divide last 3 digits by 8
9	Add up the digits and if the sum is divisible by 9 then so is the original number
10	Ends in 0

Example 1

What divides evenly into 3495?

Number	Check it!	Yes or No?
2	Is 3495 even?	no
3	Add the digits: 3 + 4 + 9 + 5 = 21 and 21 divides 3 evenly	yes
4	Can 95 divide 4 evenly?	no
5	Ends in 0 or 5?	yes
6	Divisible by both 2 AND 3?	no
8	Can 495 divide 8 evenly?	no
9	Add the digits: 3 + 4 + 9 + 5 = 21 and 21 does not divide 9 evenly	no
10	Ends in 0?	no

Lesson 5.3 - Practice Problems

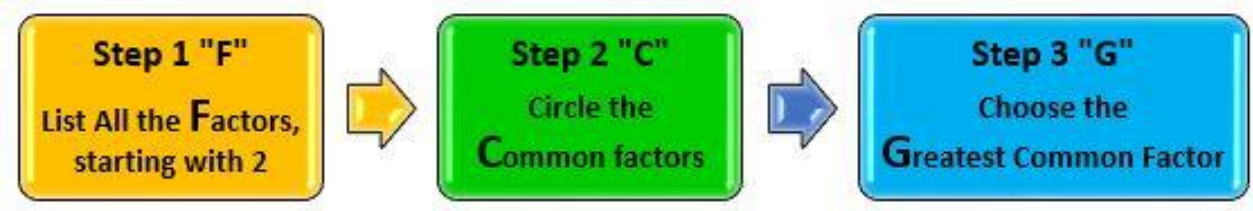
Test if the numbers in the left column are divisible by the numbers in the top row. Put a "X" in the box where divisibility holds true. Show any work below the table.

	2	3	4	5	6	8	9	10
67820								
512								
49								
3463								

Lesson 5.4 - The Greatest Common Factor

Definition:

The Greatest Common Factor (GCF) of two or more whole numbers is the largest number that is a factor of the two or more numbers. To find the GCF, we **work backwards**, following the three steps below.



Example 1

Find the GCF of 54 and 60

Step 1: List all the Factors, starting with 2

54 → 2, 3, 6, 9, 18, 27, 54

60 → 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

Step 2: Circle the Common factors

54 → 2, 3, 6, 9, 18, 27, 54

60 → 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

Step 3: Choose the Greatest circled pair

Answer: GCF (54, 60) = 6

Example 2

Write the Simplest Form of the Fraction using the GCF

$$\frac{140}{165} = ?$$

GCF (140, 165) = 5

Divide top and bottom by 5

$$\frac{140}{165} = \frac{140 \div 5}{165 \div 5} = \frac{28}{33}$$

Partner Activity 1

A forest ranger needs to remove three tree trucks by cutting the trunks into equal lengths. If the lengths of the tree trunks are six feet, eight feet and 12 feet, what is the length of the longest log that can be cut?

Partner Activity 2

Rhys wants to organize his sports cards in packets for each type of sport. Each packet has the same number of cards. If he has 24 baseball cards, 60 hockey cards, and 48 football cards, find the greatest number of cards in each packet.

Lesson 5.4 - Practice Problems

Find the Greatest Common Factor (GCF).

1. 40 and 90

2. 75 and 25

3. 168 and 85

4. 90, 120, and 150

5. 135, 225, and 405

Lesson 5.5 - The Least Common Multiple

Definition:

The smallest number that is a multiple of all numbers, excluding zero. To find the LCM, we **work backwards**, following the three steps below.

Example 1

Find the LCM of 54 and 30

Step 1: List the Multiples

54 → 54, 108, 162, 216, 270, 324,…

30 → 30, 60, 90, 120, 150, 180, 210, 240, 270, 300,…

Step 2: Circle the Common multiple(s)

54 → 54, 108, 162, 216, 270, 324,…

30 → 30, 60, 90, 120, 150, 180, 210, 240, 270, 300,…

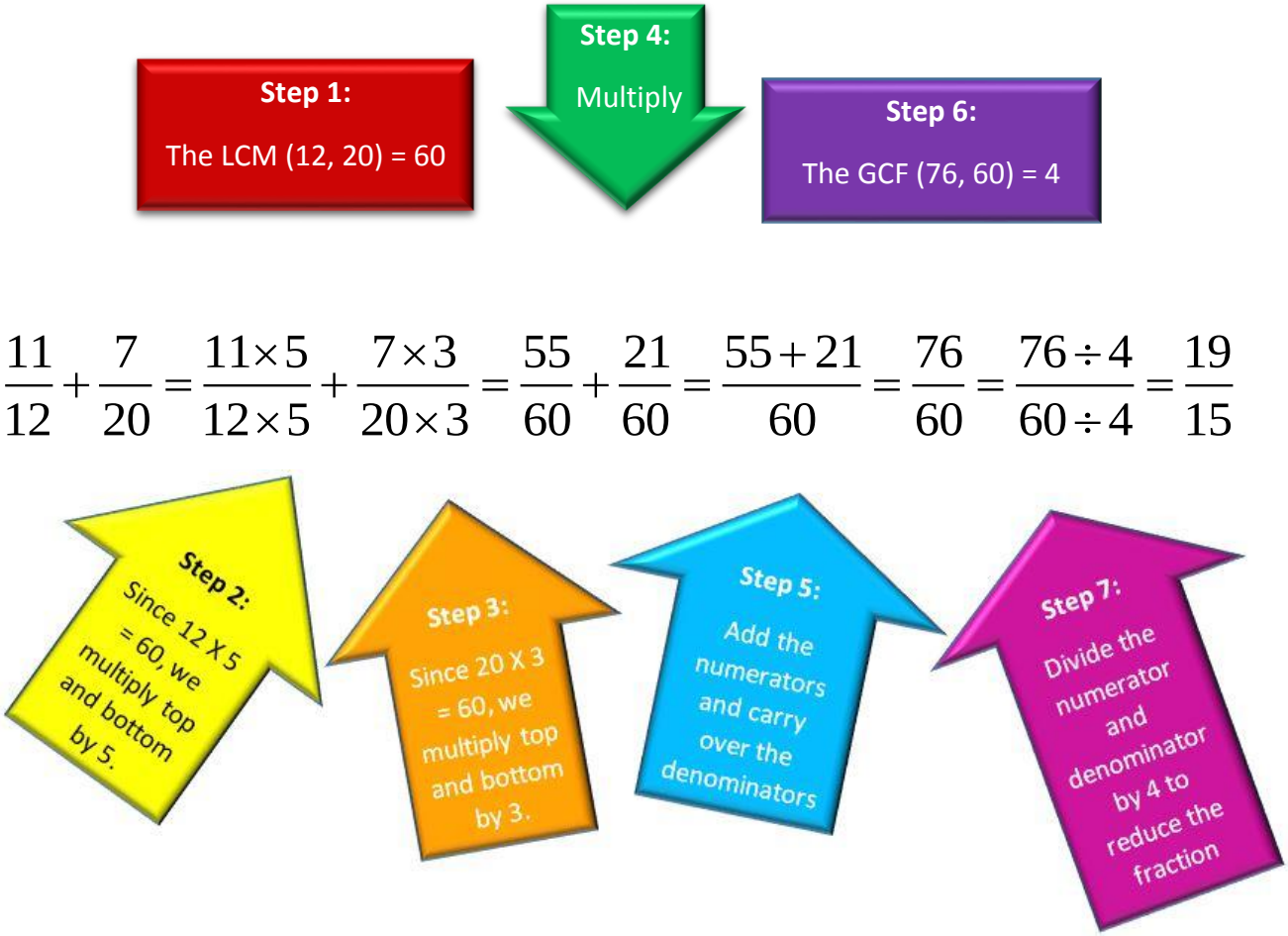
Step 3: Choose the Least common multiple = 270

Partner Activity 1

A recipe for peanut butter cookies will make 15 cookies. A recipe for chocolate cookies will make two dozen cookies. If you want to have the same number of each type of cookie, what is the least number of each that you will need to make using complete recipes?

Example 2

Add (and Subtract) Fractions Using the LCM and GCF



$$\frac{11}{12}+\frac{7}{20}=\frac{11\times5}{12\times5}+\frac{7\times3}{20\times3}=\frac{55}{60}+\frac{21}{60}=\frac{55+21}{60}=\frac{76}{60}=\frac{76\div4}{60\div4}=\frac{19}{15}$$

Partner Activity 2

Add or subtract the following fractions:

1. $\frac{5}{6}-\frac{3}{4}$
2. $\frac{2}{3}-\frac{1}{2}$
3. $6\frac{4}{5}+7\frac{1}{5}$

Lesson 5.5 - Practice Problems

Find the Least Common Multiple (LCM).

1. 40 and 90

2. 75 and 25

3. 168 and 85

4. 90, 120, and 150

5. 135, 225, and 405

Add or Subtract fractions.

6. $\frac{7}{12}+\frac{8}{5}$

7. $\frac{3}{16}-\frac{2}{9}$

8. $\frac{34}{26}+\frac{10}{13}$

Extension: Methods of Teaching Mathematics

Part 1

Using the standard and topic you choose from earlier this semester, write a full 45-minute lesson plan. See Canvas for more detailed instructions.

Part 2

Make sure you are working on Khan Academy throughout the semester.

6 - Geometry

Lesson 6.1 - The Why

The Essential Questions

BEFORE YOU BEGIN CHAPTER 6, ANSWER THE ESSENTIAL QUESTIONS HERE:

Why are Teachers Learning this Material?

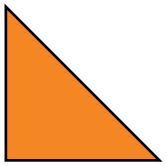
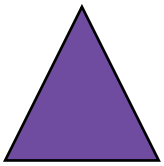
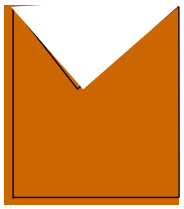
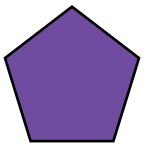
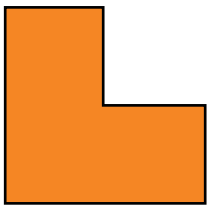
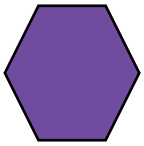
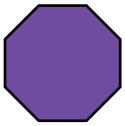
Many people study Geometry twice in their life. The first is in elementary school and the second is one year in high school. Therefore, by the time a person is ready to class on teaching mathematics it has already been many years since any formal type of Geometry has been practiced in a classroom setting.

Why are Elementary School Students Learning this Mathematics?

Geometry is a unique branch of mathematics, which gives students a real-life (outside of the classroom) approach to mathematics. The students learn spatial reasoning and problem solving skills. From kindergarten to 6^{th} grade, students learn about shapes and solids.

Lesson 6.2 - Polygons

Below is a table of polygons. There are an infinite amount of polygons, but the following are the shapes taught in elementary school.

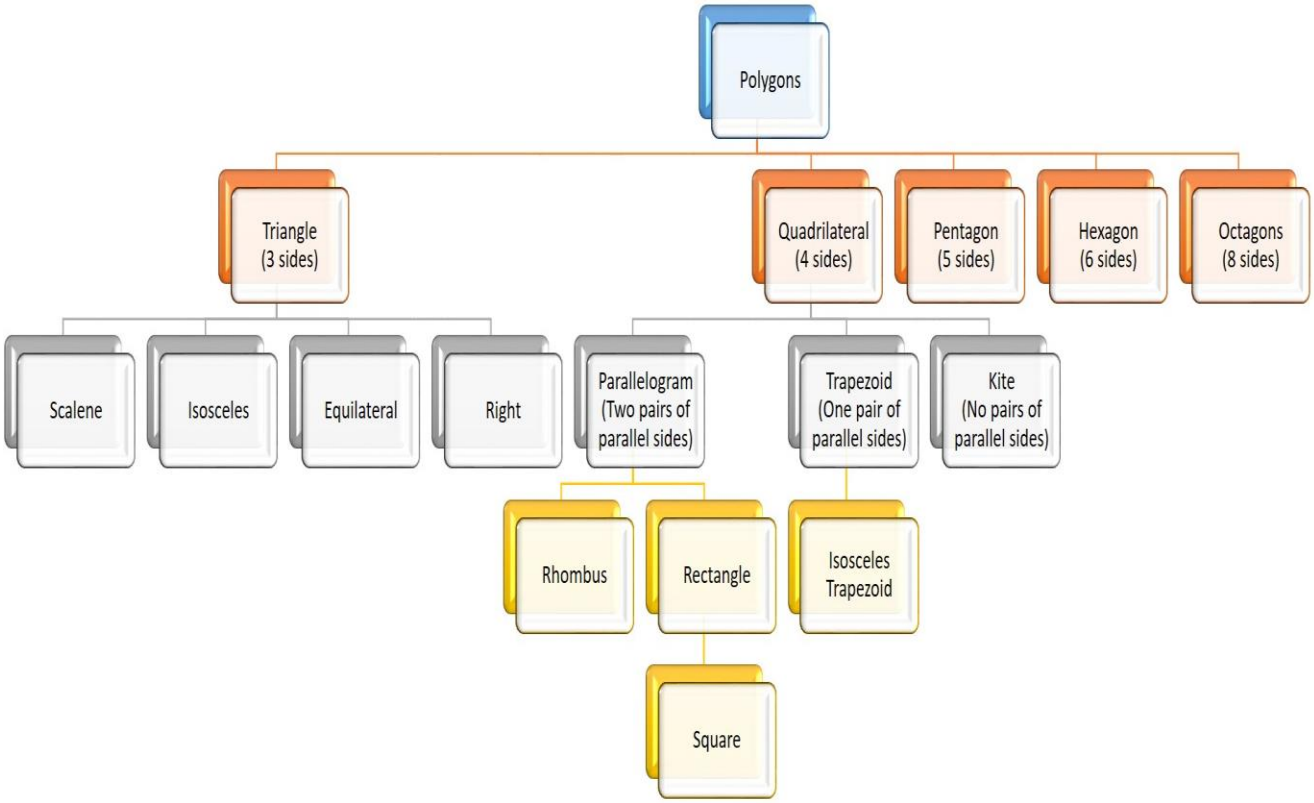
Number of Sides	Name	Irregular Polygon	Regular Polygon
3 sides	Triangle		
4 sides	Quadrilateral		
5 sides	Pentagon		
6 sides	Hexagon		
8 sides	Octagon		

Definitions:

Regular Polygon: a shape whose sides have the same length and whose angles have the same measure

Irregular Polygon: a shape whose sides differ in length or have angles of different measure

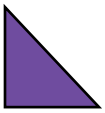
Hierarchy of Polygons



Polygon Definitions

- **Kite:** a quadrilateral with two consecutive sides having equal lengths and the other two sides also have equal lengths
- **Trapezoid:** a quadrilateral with at least one pair of opposite sides parallel
- **Isosceles Trapezoid:** a trapezoid with both angles next to one of the parallel sides having the same size
- **Parallelogram:** a trapezoid with pairs of opposite sides parallel
- **Rectangle:** a parallelogram with a right angle
- **Rhombus:** a quadrilateral with all sides being the same
- **Square:** a rectangle that has four equal sides

Types of Triangles

Name	Definition	Triangle
SIDES		
Equilateral	All three sides are equal	
Isosceles	Only two sides are equal	
Scalene	All three sides are different in length	
ANGLES		
Acute	Each angle is less than 90^0	
Right	One angle is 90^0	
Obtuse	One angle is more than 90^0	

Partner Activity 1

Draw the following triangles

1. Isosceles right triangle

2. Scalene obtuse triangle

3. Equilateral right triangle

Partner Activity 2

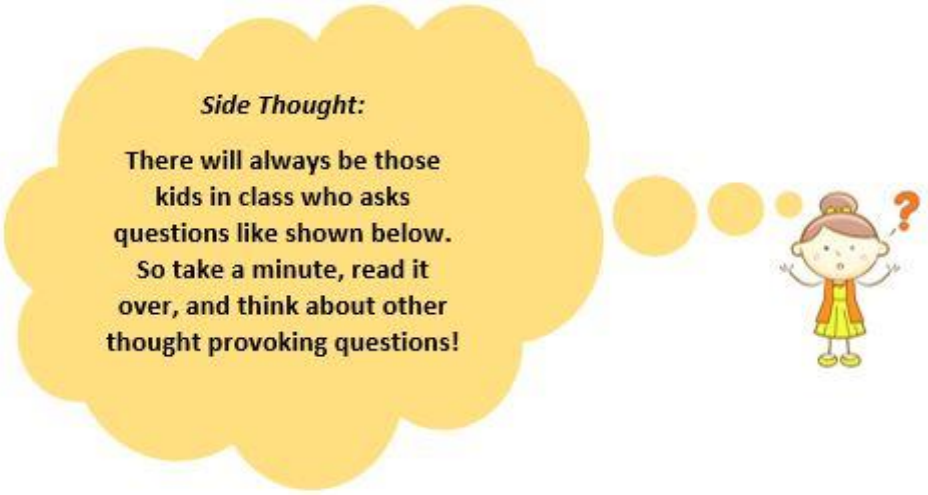
1. Is a rectangle a square? Is a square a rectangle?

2. *Multiple Choice*: Which one is NOT a name for the figure below?

   a. Polygon
   b. Quadrilateral
   c. Parallelogram
   d. Trapezoid

3. What is the difference between a regular and irregular polygon?

Facts about Angles



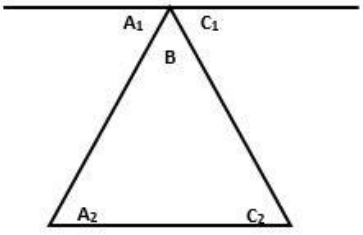
- Angles in a triangle add up to 180^0
- An angle forming a straight line is also 180^0
- Any quadrilateral (4-sided figure) is 360^0
- Angles which round a point add up to 360^0
- The two base angles of an isosceles triangle are equal

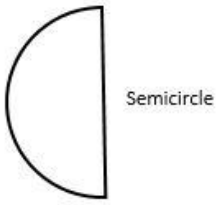
Why does a triangle add up to 180^o?

A full circle is 360^o. Half of a circle, called a semicircle, would then be 180^0. The diameter (a line which passes through the center of the circle) of the semicircle is then also 180^0. Therefore, all straight lines are 180^o. See the figure below.

Knowing that all straight lines are 180^o, we look at the figure below of the line and triangle. Since a line is 180^o, we know that angles A_1, B, and C_1 must add up to 180^o. A theorem (proven statement) in Geometry states that alternate (opposite sides) interior angles are congruent (equal). Angles A_1 and A_2 are alternate interior, cut by the transversal (line) connecting angle A_2 to the straight line. Angles C_1 and C_2 follow a similar approach.

Since the measures of angles $A_1 = A_2$, $C_1 = C_2$, and $A_1 + B + C_1 = 180$, then by substitution, $A_2 + B + C_2 = 180$. Therefore, triangle A_2BC_2 adds up to 180^o.





Partner Activity 3

The sum of the interior angles of any polygon is represented by: $180(n-2)$.

1. Find the sum of the interior angles of a triangle, using the formula.

2. Find the sum of the interior angles of a pentagon, using the formula.

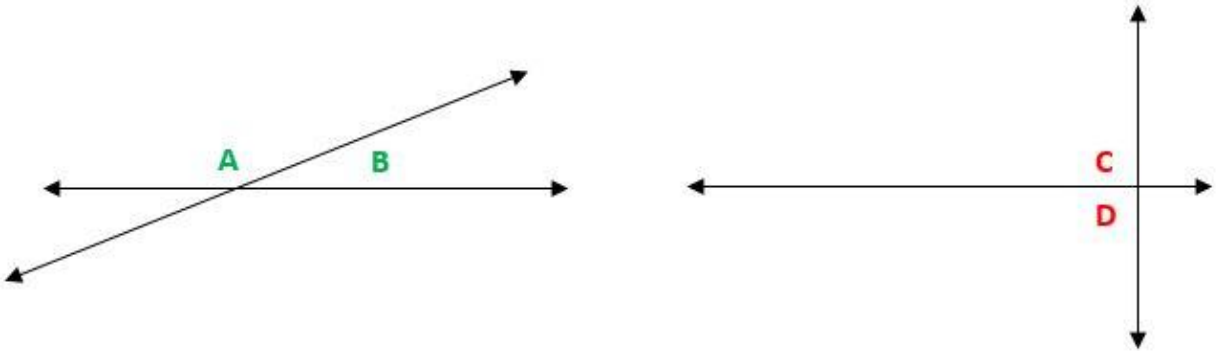
3. Find the sum of the interior angles of a 15-sided polygon, using the formula.

4. What is the sum of the EXTERIOR angles of a pentagon?

Complementary and Supplementary Angles

Definitions:

- **Complementary angles** are any two angles with a sum of 90^0. See angles **C** and **D** below.
- **Supplementary angles** are any two angles with a sum of 180^0. See angles **A** and **B** below.



Partner Activity 4

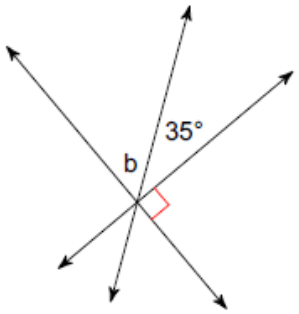
1. You have two supplementary angles. One angle is 30^0. What is the measure of the other angle?

2. One angle is complementary to another angle. The first one is 49^0. What is the measure of the second angle?

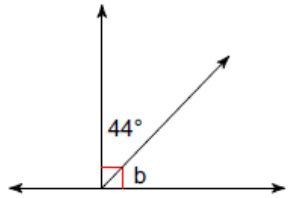
Lesson 6.2 - Practice Problems

(Problems 1 – 4) Find the measure of angle b.

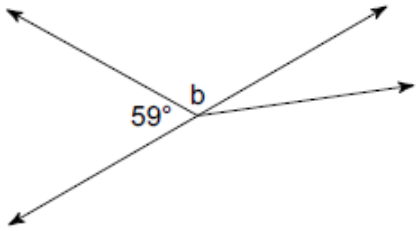
1.



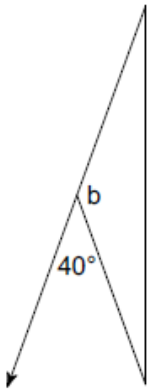
2.



3.

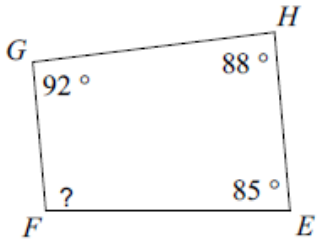


4.

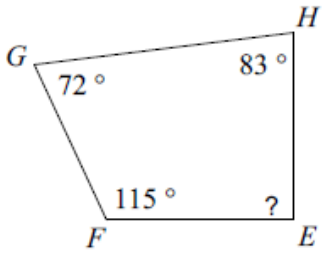


(Problems 5 – 6) Find the measure of each angle indicated.

5.



6.



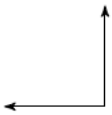
(Problems 7 – 10) Classify each angle as acute, obtuse, right or straight.

7. 121^{o}

8. 180^{o}

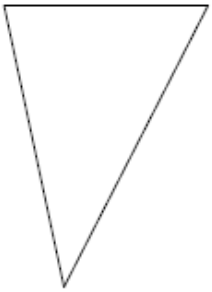
9.

10.

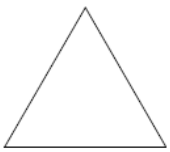


(Problems 11 – 12) Classify each triangle by its angles.

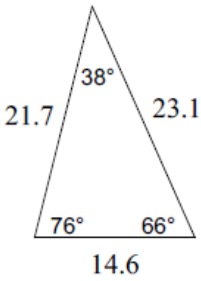
11.



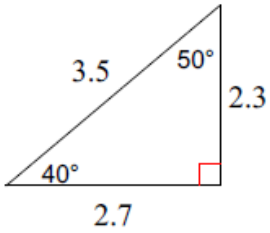
12.



(Problems 13 – 14) Classify each triangle by its angles and sides.



13.



14.

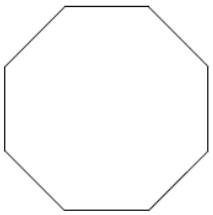
(Problems 15 – 16) Sketch an example of the type of triangle described.

15. Acute Isosceles

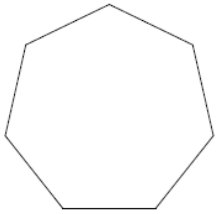
16. Right Obtuse

(Problems 17 – 18) Write the name of each polygon.

17.

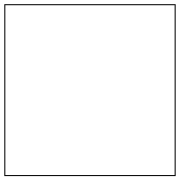


18.

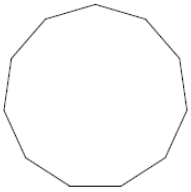


(Problems 19 – 22) Find the interior angle sum for each polygon. Round your answer to the nearest tenth, if necessary.

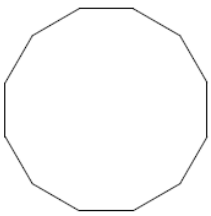
19.

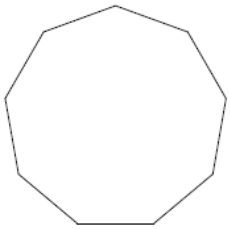


20.



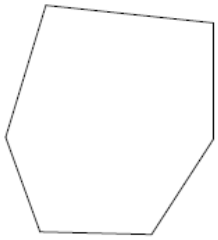
21.



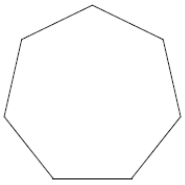


22.

(Problems 23 – 26) State if the polygon is regular or irregular.



23.



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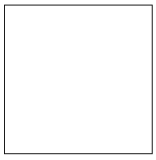
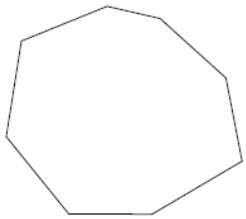


25.



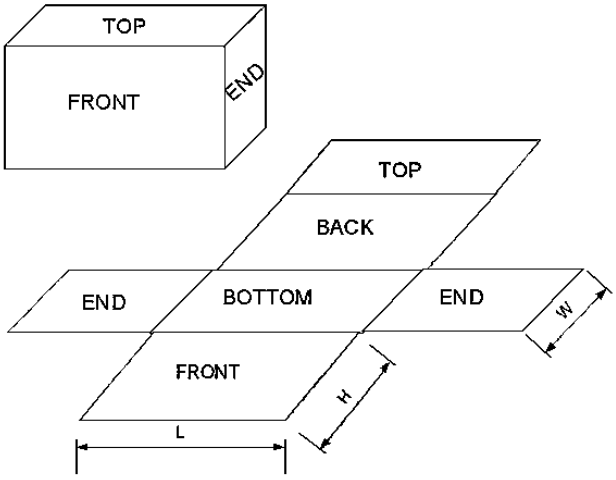
26.

Lesson 6.3 - Area, Surface Area and Volume

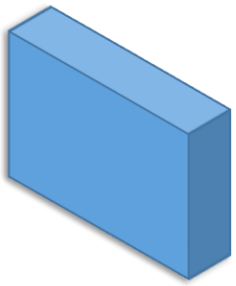
Definitions

1. **Area**: the extent or measurement of a surface or piece of land. (2 dimensional)

2. **Surface Area**: the area of such an outer part or uppermost layer. (3 dimensional)



3. **Volume**: the amount of space that a substance or object occupies, or that is enclosed within a container, especially when great. (3 dimensional)



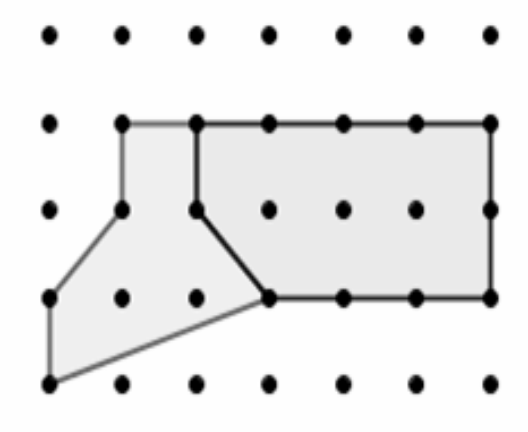
Partner Activity 1

1. Why is area "squared"? i.e. $15cm^2$

2. Why is volume "cubed"? i.e. 40 $liters^3$

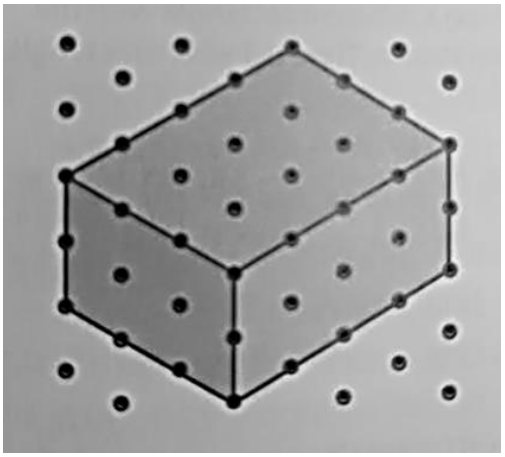
Partner Activity 2

Think *inside* the box and approximate the Shaded Area: (area of a square is base times height)



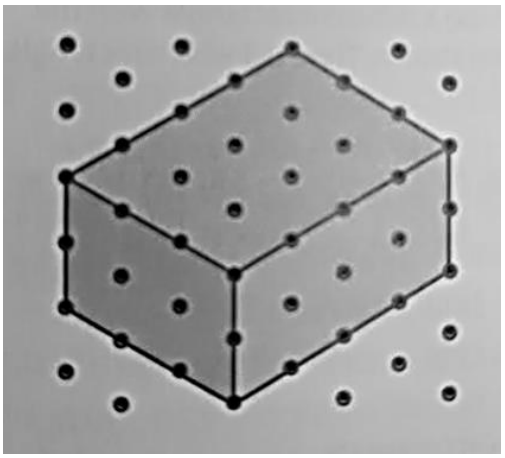
Partner Activity 3

Think *around* the box (surface area) and approximate the Shaded Area: (How many sides are not seen in the picture, which must be included in the final answer?)



Partner Activity 4

Think *inside* the box (volume) and approximate the Shaded Area: Volume is base time's height times width.

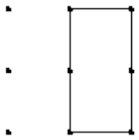


Lesson 6.3 - Practice Problems

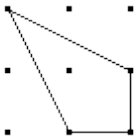
1. Explain the difference between area, surface area, and volume.

2. Estimate the area of the following shapes:

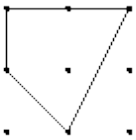
a.



b.

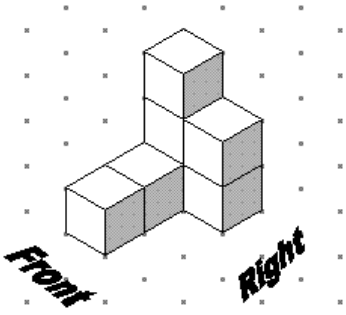


c.



3. Find the Surface Area of the following shapes:

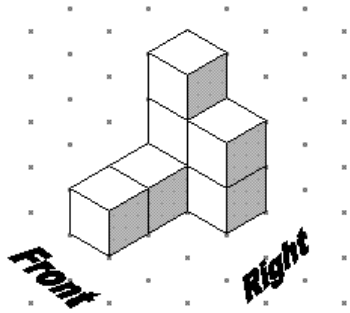
a.



b.

4. Find the Volume of the following shapes:

a.



b.

Lesson 6.4 - Linear Unit Conversions

1. Convert 2 miles into _____ feet.

Answer: Since 1 mile = 5280 feet. Therefore 2 miles = 2(5280 feet) = **10560 feet**

2. Convert 15 yards into _____ miles.

Answer: Since 1 mile = 5280 feet and 3 feet = 1 yard. Follow the math below:

$$\frac{15\ yards}{1}\times\frac{3\ feet}{1\ yard}\times\frac{1\ mile}{5280\ feet}=$$

$$\frac{15\ \cancel{yards}}{1}\times\frac{3\ \cancel{feet}}{1\ \cancel{yard}}\times\frac{1\ mile}{5280\ \cancel{feet}}=\frac{45}{5280}miles$$

Partner Activity 1

1. Convert 20 inches into _____ yards.

2. Convert 16 miles into _____ feet.

Lesson 6.4 - Practice Problems

1. Convert 42 feet into ____ miles.

2. Convert 81 inches into _____ yards.

3. Convert 34 miles into _____ yards.

4. Convert 91 yards into _____ inches.

Lesson 6.5 - Area, Surface Area and Volume Formulas

Area formulas:

Let b = base

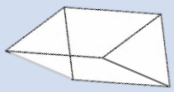
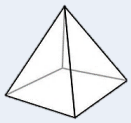
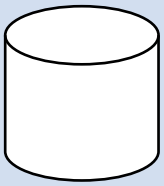
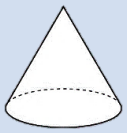
Let h = height

Let s = side

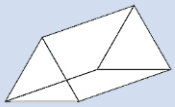
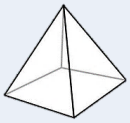
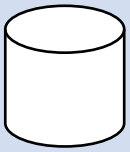
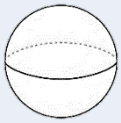
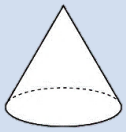
Let r = radius

Shape Name	Shape	Area Formula
Rectangle		$A = bh$
Square		$A = bh$ $A = s^2$
Parallelogram		$A = bh$
Triangle		$A = \frac{1}{2}bh$
Circle		$A = \pi r^2$
Trapezoid		$A = \frac{1}{2}h(b_1 + b_2)$

Surface Area Formulas

Geometric Figure	Surface Area Formula	Surface Area Meaning	Variables
Prism 	$SA = 2B + Ph$	Find the area of each face. Add up all areas.	SA = Surface Area B = area of the base of the figure P = perimeter of the base of the figure h = height s = slant height r = radius
Pyramid 	$SA = B + \frac{1}{2}sP$	Find the area of each face. Add up all areas.	
Cylinder 	$SA = 2B + 2\pi rh$	Find the area of the base, times 2, then add the areas to the areas of the rectangle, which is the circumference times the height.	
Sphere 	$SA = 4\pi r^2$	Find the area of the great circle and multiply it by 4.	
Cone 	$SA = B + \pi rs$	Find the area of the base and add the product of the radius times the slant height times PI.	

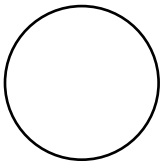
Volume Formulas

Geometric Figure	Volume Formula	Volume Meaning	Variables
Prism 	$V = Bh$	Find the area of the base and multiply it by the height	SA = Surface Area B = area of the base of the figure P = perimeter of the base of the figure h = height s = slant height r = radius
Pyramid 	$V = \frac{1}{3}Bh$	Find the area of the base and multiply it by 1/3 of the height.	
Cylinder 	$V = Bh$	Find the area of the base and multiply it by the height.	
Sphere 	$V = \frac{4}{3}\pi r^3$	Find the area of the great circle and multiply it by the radius and then multiply it by $\frac{4}{3}$.	
Cone 	$V = \frac{1}{3}Bh$	Find the area of the base and multiply it by $\frac{1}{3}$ of the height.	

Color Coding:
For Examples 1 – 5:
The **formula** is **blue**, the **exact answer** is **red** and the **rounded, decimal answer** is **green**.

Example 1

Find the area of a circle with diameter of 14 feet.



$$A = \pi r^2 = \pi(7)^2 = 49\pi \text{ feet}^2 = 153.86 \text{ feet}^2$$

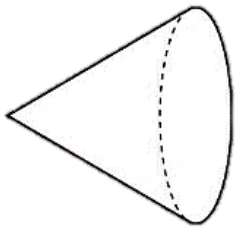
Example 2

Find the area of a trapezoid with a height of 12 inches, and bases of 24 and 10 inches.

$$A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(12)(24+10) = 6(34) = 204 \text{ inches}^2$$

Example 3

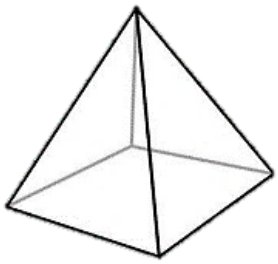
Find the surface area of a cone with a slant height of 8 cm and a radius of 3 cm.



$$SA = B + \pi rs = (\pi r^2) + \pi rs = (\pi(3^2)) + \pi(3)(8)$$
$$= 9\pi + 24\pi = 33\pi \text{ cm}^2 = 103.62 \text{ cm}^2$$

Example 4

Find the surface area of a rectangular pyramid with a slant height of 10 yards, a base width (b) of 8 yards and a base length (h) of 12 yards.



$$SA = B + \frac{1}{2}sP = (bh) + \frac{1}{2}s(2b + 2h)$$

$$= (8)(12) + \frac{1}{2}(10)(2(8) + 2(12))$$

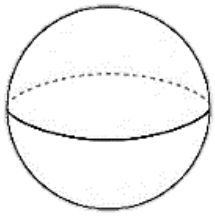
$$= 96 + \frac{1}{2}(10)(16 + 24)$$

$$= 96 + 5(40)$$

$$= 296 \text{ yards}^2$$

Example 5

Find the volume of a sphere with a diameter of 6 meters.



$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(3)^3 = \frac{4}{3}(27\pi) = 36\pi \text{ meters}^3 = 113.04 \text{ meters}^3$$

Partner Activity 1

1. Find the area of a triangle with a base of 40 inches and a height of 60 inches.

2. Find the area of a square with a side of 15 feet.

3. Find the surface area of Earth, which has a diameter of 7917.5 miles. Use 3.14 for PI.

4. Find the volume of a can a soup, which has a radius of 2 inches and a height of 3 inches. Use 3.14 for PI.

Lesson 6.5 - Practice Problems

(Problems 1 – 4) Find the area of each circle with the given parameters. Use 3.14 for PI. Round your answer to the nearest tenth.

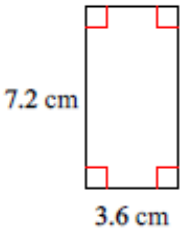
1. Radius = 9 cm

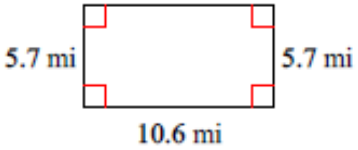
2. Diameter = 6 miles

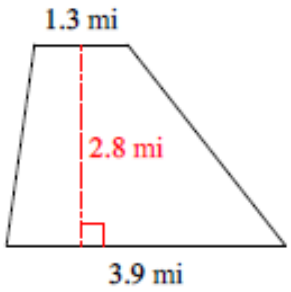
3. Radius = 8.6 cm

4. Diameter = 14 meters

(Problems 5 – 8) Find the area of each polygon. Round answers to the nearest tenth.

5. 

6. 



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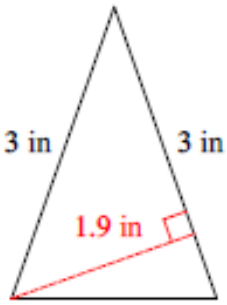
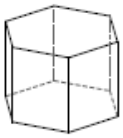


8.

(Problems 9 – 12) Name each figure.

9.



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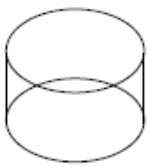
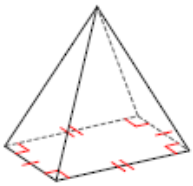
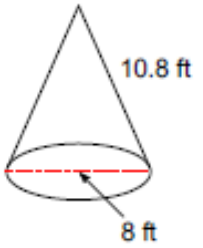


11.

12.



(Problems 13 – 17) Find the surface area of each figure. Leave your answers in terms of PI, if the answer contains PI. Round all other answers to the nearest hundredth.



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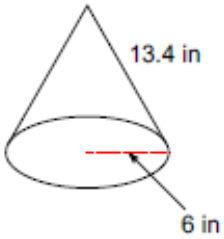

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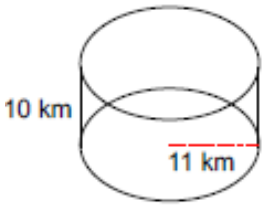
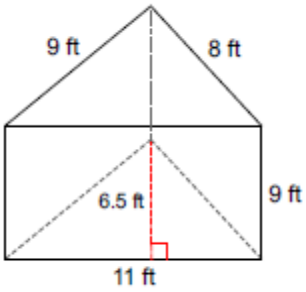
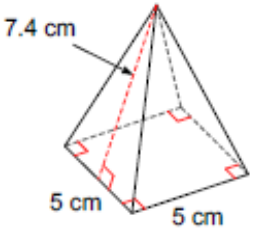


15.

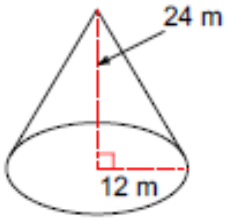


16.

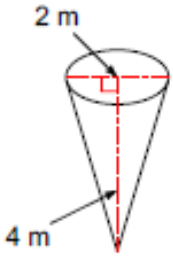


17.

(Problems 18 – 25) Find the volume of each figure. Leave your answers in terms of PI, for answers that contain PI. Round all other answers to the nearest hundredth.



18.



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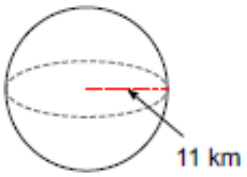

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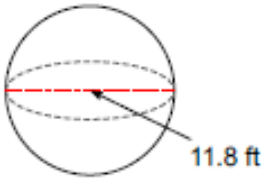

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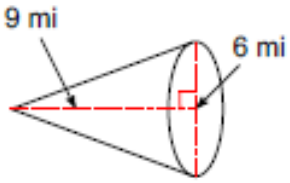

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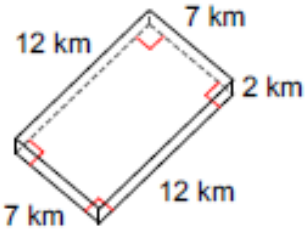

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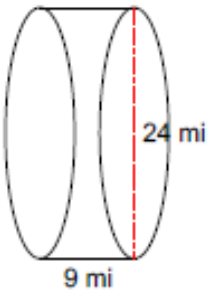

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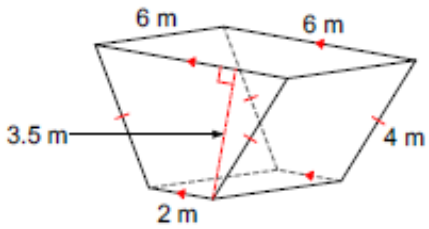


25.

Extension: Methods of Teaching Mathematics

Part 1

Assessments:

1. What is the Difference between Formative and Summative Assessments? Which One is More Important?
2. Formative Assessment Examples and When to Use Them
3. Summative Assessment Examples and When to Use Them

Part 2

Write a Formative and Summative Assessment for Your Lesson Plan

Part 3

Make sure you are working on Khan Academy throughout the semester.

Appendix A

The California Common Core State Standards, Mathematics, K Through 6

Decoding the Standards: CCSS.MATH.CONTENT.K.CC.B.4.C

Common Core State Standards. Mathematics. Content. Kindergarten. Counting and Cardinality. B = second group in Counting and Cardinality. 4 = 4th standard listed in Counting and Cardinality. C = third part of standard B.4

Kindergarten

Counting & Cardinality

Know number names and the count sequence.

CCSS.MATH.CONTENT.K.CC.A.1
Count to 100 by ones and by tens.

CCSS.MATH.CONTENT.K.CC.A.2
Count forward beginning from a given number within the known sequence (instead of having to begin at 1).

CCSS.MATH.CONTENT.K.CC.A.3
Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).

Count to tell the number of objects.

CCSS.MATH.CONTENT.K.CC.B.4
Understand the relationship between numbers and quantities; connect counting to cardinality.

CCSS.MATH.CONTENT.K.CC.B.4.A
When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.

CCSS.MATH.CONTENT.K.CC.B.4.B
Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.

CCSS.MATH.CONTENT.K.CC.B.4.C
Understand that each successive number name refers to a quantity that is one larger.

CCSS.MATH.CONTENT.K.CC.B.5
Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.

Compare numbers.

CCSS.MATH.CONTENT.K.CC.C.6
Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[1]

CCSS.MATH.CONTENT.K.CC.C.7
Compare two numbers between 1 and 10 presented as written numerals.

Operations & Algebraic Thinking

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

CCSS.MATH.CONTENT.K.OA.A.1
Represent addition and subtraction with objects, fingers, mental images, drawings[1], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

CCSS.MATH.CONTENT.K.OA.A.2
Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

CCSS.MATH.CONTENT.K.OA.A.3
Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

CCSS.MATH.CONTENT.K.OA.A.4
For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

CCSS.MATH.CONTENT.K.OA.A.5
Fluently add and subtract within 5.

Number & Operations in Base Ten

Work with numbers 11-19 to gain foundations for place value.

CCSS.MATH.CONTENT.K.NBT.A.1
Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Measurement & Data

Describe and compare measurable attributes.

CCSS.MATH.CONTENT.K.MD.A.1
Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.

CCSS.MATH.CONTENT.K.MD.A.2
Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter*.

Classify objects and count the number of objects in each category.

CCSS.MATH.CONTENT.K.MD.B.3
Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.

Geometry

Identify and describe shapes.

CCSS.MATH.CONTENT.K.G.A.1
Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*.

CCSS.MATH.CONTENT.K.G.A.2
Correctly name shapes regardless of their orientations or overall size.

CCSS.MATH.CONTENT.K.G.A.3
Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").

Analyze, compare, create, and compose shapes.

CCSS.MATH.CONTENT.K.G.B.4
Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).

CCSS.MATH.CONTENT.K.G.B.5
Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.

CCSS.MATH.CONTENT.K.G.B.6
Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle*?"

Grade 1

Operations & Algebraic Thinking

Represent and solve problems involving addition and subtraction.

CCSS.MATH.CONTENT.1.OA.A.1
Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

CCSS.MATH.CONTENT.1.OA.A.2
Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Understand and apply properties of operations and the relationship between addition and subtraction.

CCSS.MATH.CONTENT.1.OA.B.3
Apply properties of operations as strategies to add and subtract.[2] *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)*

CCSS.MATH.CONTENT.1.OA.B.4
Understand subtraction as an unknown-addend problem. *For example, subtract 10 - 8 by finding the number that makes 10 when added to 8.*

Add and subtract within 20.

CCSS.MATH.CONTENT.1.OA.C.5
Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).

CCSS.MATH.CONTENT.1.OA.C.6
Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 - 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

Work with addition and subtraction equations.

CCSS.MATH.CONTENT.1.OA.D.7
Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 - 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.

CCSS.MATH.CONTENT.1.OA.D.8
Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = _ - 3, 6 + 6 = _.*

Number & Operations in Base Ten

Extend the counting sequence.

CCSS.MATH.CONTENT.1.NBT.A.1
Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

Understand place value.

CCSS.MATH.CONTENT.1.NBT.B.2
Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:

CCSS.MATH.CONTENT.1.NBT.B.2.A
10 can be thought of as a bundle of ten ones — called a "ten."

CCSS.MATH.CONTENT.1.NBT.B.2.B
The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.

CCSS.MATH.CONTENT.1.NBT.B.2.C
The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

CCSS.MATH.CONTENT.1.NBT.B.3
Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

Use place value understanding and properties of operations to add and subtract.

CCSS.MATH.CONTENT.1.NBT.C.4
Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain

the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

CCSS.MATH.CONTENT.1.NBT.C.5
Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.

CCSS.MATH.CONTENT.1.NBT.C.6
Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Measurement & Data

Measure lengths indirectly and by iterating length units.

CCSS.MATH.CONTENT.1.MD.A.1
Order three objects by length; compare the lengths of two objects indirectly by using a third object.

CCSS.MATH.CONTENT.1.MD.A.2
Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.

Tell and write time.

CCSS.MATH.CONTENT.1.MD.B.3
Tell and write time in hours and half-hours using analog and digital clocks.

Represent and interpret data.

CCSS.MATH.CONTENT.1.MD.C.4
Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

Geometry

Reason with shapes and their attributes.

CCSS.MATH.CONTENT.1.G.A.1
Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.

CCSS.MATH.CONTENT.1.G.A.2
Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.

CCSS.MATH.CONTENT.1.G.A.3
Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

Grade 2

Operations & Algebraic Thinking

Represent and solve problems involving addition and subtraction.

CCSS.MATH.CONTENT.2.OA.A.1
Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Add and subtract within 20.

CCSS.MATH.CONTENT.2.OA.B.2
Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.

Work with equal groups of objects to gain foundations for multiplication.

CCSS.MATH.CONTENT.2.OA.C.3
Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

CCSS.MATH.CONTENT.2.OA.C.4
Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Number & Operations in Base Ten

Understand place value.

CCSS.MATH.CONTENT.2.NBT.A.1
Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

CCSS.MATH.CONTENT.2.NBT.A.1.A
100 can be thought of as a bundle of ten tens — called a "hundred."

CCSS.MATH.CONTENT.2.NBT.A.1.B
The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

CCSS.MATH.CONTENT.2.NBT.A.2
Count within 1000; skip-count by 5s, 10s, and 100s.

CCSS.MATH.CONTENT.2.NBT.A.3
Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

CCSS.MATH.CONTENT.2.NBT.A.4
Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

Use place value understanding and properties of operations to add and subtract.

CCSS.MATH.CONTENT.2.NBT.B.5
Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

CCSS.MATH.CONTENT.2.NBT.B.6
Add up to four two-digit numbers using strategies based on place value and properties of operations.

CCSS.MATH.CONTENT.2.NBT.B.7
Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

CCSS.MATH.CONTENT.2.NBT.B.8
Mentally add 10 or 100 to a given number 100-900, and mentally subtract 10 or 100 from a given number 100-900.

CCSS.MATH.CONTENT.2.NBT.B.9
Explain why addition and subtraction strategies work, using place value and the properties of operations.

Measurement & Data

Measure and estimate lengths in standard units.

CCSS.MATH.CONTENT.2.MD.A.1
Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

CCSS.MATH.CONTENT.2.MD.A.2
Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.

CCSS.MATH.CONTENT.2.MD.A.3
Estimate lengths using units of inches, feet, centimeters, and meters.

CCSS.MATH.CONTENT.2.MD.A.4
Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Relate addition and subtraction to length.

CCSS.MATH.CONTENT.2.MD.B.5
Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.

CCSS.MATH.CONTENT.2.MD.B.6
Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

Work with time and money.

CCSS.MATH.CONTENT.2.MD.C.7
Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.

CCSS.MATH.CONTENT.2.MD.C.8
Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have?

Represent and interpret data.

CCSS.MATH.CONTENT.2.MD.D.9
Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.

CCSS.MATH.CONTENT.2.MD.D.10
Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems[1] using information presented in a bar graph.

Geometry

Reason with shapes and their attributes.

CCSS.MATH.CONTENT.2.G.A.1
Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

CCSS.MATH.CONTENT.2.G.A.2
Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

CCSS.MATH.CONTENT.2.G.A.3
Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

Grade 3

Operations & Algebraic Thinking

Represent and solve problems involving multiplication and division.

CCSS.MATH.CONTENT.3.OA.A.1
Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7*.

CCSS.MATH.CONTENT.3.OA.A.2
Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8*.

CCSS.MATH.CONTENT.3.OA.A.3
Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]

CCSS.MATH.CONTENT.3.OA.A.4
Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?*

Understand properties of multiplication and the relationship between multiplication and division.

CCSS.MATH.CONTENT.3.OA.B.5
Apply properties of operations as strategies to multiply and divide.[2] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*

CCSS.MATH.CONTENT.3.OA.B.6
Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8*.

Multiply and divide within 100.

CCSS.MATH.CONTENT.3.OA.C.7
Fluently multiply and divide within 100, using strategies such as the relationship

between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

CCSS.MATH.CONTENT.3.OA.D.8
Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.[3]

CCSS.MATH.CONTENT.3.OA.D.9
Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends*.

Number & Operations in Base Ten

Use place value understanding and properties of operations to perform multi-digit arithmetic.

CCSS.MATH.CONTENT.3.NBT.A.1
Use place value understanding to round whole numbers to the nearest 10 or 100.

CCSS.MATH.CONTENT.3.NBT.A.2
Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

CCSS.MATH.CONTENT.3.NBT.A.3
Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

Number & Operations—Fractions

Develop understanding of fractions as numbers.

CCSS.MATH.CONTENT.3.NF.A.1
Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*.

CCSS.MATH.CONTENT.3.NF.A.2

Understand a fraction as a number on the number line; represent fractions on a number line diagram.

CCSS.MATH.CONTENT.3.NF.A.2.A

Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.

CCSS.MATH.CONTENT.3.NF.A.2.B

Represent a fraction *a*/*b* on a number line diagram by marking off a lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line.

CCSS.MATH.CONTENT.3.NF.A.3

Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

CCSS.MATH.CONTENT.3.NF.A.3.A

Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

CCSS.MATH.CONTENT.3.NF.A.3.B

Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.

CCSS.MATH.CONTENT.3.NF.A.3.C

Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram*.

CCSS.MATH.CONTENT.3.NF.A.3.D

Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Measurement & Data

Solve problems involving measurement and estimation.

CCSS.MATH.CONTENT.3.MD.A.1

Tell and write time to the nearest minute and measure time intervals in minutes. Solve

word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

CCSS.MATH.CONTENT.3.MD.A.2
Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[2]

Represent and interpret data.

CCSS.MATH.CONTENT.3.MD.B.3
Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets*.

CCSS.MATH.CONTENT.3.MD.B.4
Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

CCSS.MATH.CONTENT.3.MD.C.5
Recognize area as an attribute of plane figures and understand concepts of area measurement.

> CCSS.MATH.CONTENT.3.MD.C.5.A
> A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.
>
> CCSS.MATH.CONTENT.3.MD.C.5.B
> A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.

CCSS.MATH.CONTENT.3.MD.C.6
Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

CCSS.MATH.CONTENT.3.MD.C.7
Relate area to the operations of multiplication and addition.

CCSS.MATH.CONTENT.3.MD.C.7.A
Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

CCSS.MATH.CONTENT.3.MD.C.7.B
Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

CCSS.MATH.CONTENT.3.MD.C.7.C
Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.

CCSS.MATH.CONTENT.3.MD.C.7.D
Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Geometric measurement: recognize perimeter.

CCSS.MATH.CONTENT.3.MD.D.8
Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Geometry

Reason with shapes and their attributes.

CCSS.MATH.CONTENT.3.G.A.1
Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

CCSS.MATH.CONTENT.3.G.A.2
Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape*.

Grade 4

Operations & Algebraic Thinking

Use the four operations with whole numbers to solve problems.

CCSS.MATH.CONTENT.4.OA.A.1
Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

CCSS.MATH.CONTENT.4.OA.A.2
Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[1]

CCSS.MATH.CONTENT.4.OA.A.3
Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Gain familiarity with factors and multiples.

CCSS.MATH.CONTENT.4.OA.B.4
Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.

Generate and analyze patterns.

CCSS.MATH.CONTENT.4.OA.C.5
Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way*.

Number & Operations in Base Ten

Generalize place value understanding for multi-digit whole numbers.

CCSS.MATH.CONTENT.4.NBT.A.1
Recognize that in a multi-digit whole number, a digit in one place represents ten times

what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division*.

CCSS.MATH.CONTENT.4.NBT.A.2
Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

CCSS.MATH.CONTENT.4.NBT.A.3
Use place value understanding to round multi-digit whole numbers to any place.

Use place value understanding and properties of operations to perform multi-digit arithmetic.

CCSS.MATH.CONTENT.4.NBT.B.4
Fluently add and subtract multi-digit whole numbers using the standard algorithm.

CCSS.MATH.CONTENT.4.NBT.B.5
Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

CCSS.MATH.CONTENT.4.NBT.B.6
Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Number & Operations—Fractions

Extend understanding of fraction equivalence and ordering.

CCSS.MATH.CONTENT.4.NF.A.1
Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

CCSS.MATH.CONTENT.4.NF.A.2
Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Build fractions from unit fractions.

CCSS.MATH.CONTENT.4.NF.B.3
Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.

CCSS.MATH.CONTENT.4.NF.B.3.A
Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

CCSS.MATH.CONTENT.4.NF.B.3.B
Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8*.

CCSS.MATH.CONTENT.4.NF.B.3.C
Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

CCSS.MATH.CONTENT.4.NF.B.3.D
Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

CCSS.MATH.CONTENT.4.NF.B.4
Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

CCSS.MATH.CONTENT.4.NF.B.4.A
Understand a fraction a/b as a multiple of $1/b$. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4)*.

CCSS.MATH.CONTENT.4.NF.B.4.B
Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*

CCSS.MATH.CONTENT.4.NF.B.4.C
Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there*

will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

Understand decimal notation for fractions, and compare decimal fractions.

CCSS.MATH.CONTENT.4.NF.C.5
Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.[2] *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100*.

CCSS.MATH.CONTENT.4.NF.C.6
Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram*.

CCSS.MATH.CONTENT.4.NF.C.7
Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

Measurement & Data

Solve problems involving measurement and conversion of measurements.

CCSS.MATH.CONTENT.4.MD.A.1
Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...*

CCSS.MATH.CONTENT.4.MD.A.2
Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

CCSS.MATH.CONTENT.4.MD.A.3
Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor*.

Represent and interpret data.

CCSS.MATH.CONTENT.4.MD.B.4
Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection*.

Geometric measurement: understand concepts of angle and measure angles.

CCSS.MATH.CONTENT.4.MD.C.5
Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

CCSS.MATH.CONTENT.4.MD.C.5.A
An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.

CCSS.MATH.CONTENT.4.MD.C.5.B
An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.

CCSS.MATH.CONTENT.4.MD.C.6
Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

CCSS.MATH.CONTENT.4.MD.C.7
Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

Geometry

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

CCSS.MATH.CONTENT.4.G.A.1
Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

CCSS.MATH.CONTENT.4.G.A.2
Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

CCSS.MATH.CONTENT.4.G.A.3

Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Grade 5

Operations & Algebraic Thinking

Write and interpret numerical expressions.

CCSS.MATH.CONTENT.5.OA.A.1
Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

CCSS.MATH.CONTENT.5.OA.A.2
Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product*.

Analyze patterns and relationships.

CCSS.MATH.CONTENT.5.OA.B.3
Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so*.

Number & Operations in Base Ten

Understand the place value system.

CCSS.MATH.CONTENT.5.NBT.A.1
Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

CCSS.MATH.CONTENT.5.NBT.A.2
Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

CCSS.MATH.CONTENT.5.NBT.A.3
Read, write, and compare decimals to thousandths.

CCSS.MATH.CONTENT.5.NBT.A.3.A
Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000).

CCSS.MATH.CONTENT.5.NBT.A.3.B
Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

CCSS.MATH.CONTENT.5.NBT.A.4
Use place value understanding to round decimals to any place.

Perform operations with multi-digit whole numbers and with decimals to hundredths.

CCSS.MATH.CONTENT.5.NBT.B.5
Fluently multiply multi-digit whole numbers using the standard algorithm.

CCSS.MATH.CONTENT.5.NBT.B.6
Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

CCSS.MATH.CONTENT.5.NBT.B.7
Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Number & Operations—Fractions

Use equivalent fractions as a strategy to add and subtract fractions.

CCSS.MATH.CONTENT.5.NF.A.1
Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*

CCSS.MATH.CONTENT.5.NF.A.2
Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2*.

Apply and extend previous understandings of multiplication and division.

CCSS.MATH.CONTENT.5.NF.B.3
Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

CCSS.MATH.CONTENT.5.NF.B.4
Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

CCSS.MATH.CONTENT.5.NF.B.4.A
Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = (ac)/(bd).*

CCSS.MATH.CONTENT.5.NF.B.4.B
Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

CCSS.MATH.CONTENT.5.NF.B.5
Interpret multiplication as scaling (resizing), by:

CCSS.MATH.CONTENT.5.NF.B.5.A
Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

CCSS.MATH.CONTENT.5.NF.B.5.B
Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.

CCSS.MATH.CONTENT.5.NF.B.6
Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

CCSS.MATH.CONTENT.5.NF.B.7
Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[1]

CCSS.MATH.CONTENT.5.NF.B.7.A
Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3*.

CCSS.MATH.CONTENT.5.NF.B.7.B
Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4*.

CCSS.MATH.CONTENT.5.NF.B.7.C
Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

Measurement & Data

Convert like measurement units within a given measurement system.

CCSS.MATH.CONTENT.5.MD.A.1
Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

Represent and interpret data.

CCSS.MATH.CONTENT.5.MD.B.2
Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally*.

Geometric measurement: understand concepts of volume.

CCSS.MATH.CONTENT.5.MD.C.3

Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

CCSS.MATH.CONTENT.5.MD.C.3.A

A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.

CCSS.MATH.CONTENT.5.MD.C.3.B

A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

CCSS.MATH.CONTENT.5.MD.C.4

Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

CCSS.MATH.CONTENT.5.MD.C.5

Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

CCSS.MATH.CONTENT.5.MD.C.5.A

Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

CCSS.MATH.CONTENT.5.MD.C.5.B

Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

CCSS.MATH.CONTENT.5.MD.C.5.C

Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

Geometry

Graph points on the coordinate plane to solve real-world and mathematical problems.

CCSS.MATH.CONTENT.5.G.A.1

Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the

origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).

CCSS.MATH.CONTENT.5.G.A.2
Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Classify two-dimensional figures into categories based on their properties.

CCSS.MATH.CONTENT.5.G.B.3
Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.

CCSS.MATH.CONTENT.5.G.B.4
Classify two-dimensional figures in a hierarchy based on properties.

Grade 6

Ratios & Proportional Relationships

Understand ratio concepts and use ratio reasoning to solve problems.

CCSS.MATH.CONTENT.6.RP.A.1
Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*

CCSS.MATH.CONTENT.6.RP.A.2
Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[1]

CCSS.MATH.CONTENT.6.RP.A.3
Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

CCSS.MATH.CONTENT.6.RP.A.3.A
Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

CCSS.MATH.CONTENT.6.RP.A.3.B
Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*

CCSS.MATH.CONTENT.6.RP.A.3.C
Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

CCSS.MATH.CONTENT.6.RP.A.3.D
Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

The Number System

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

CCSS.MATH.CONTENT.6.NS.A.1
Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*.

Compute fluently with multi-digit numbers and find common factors and multiples.

CCSS.MATH.CONTENT.6.NS.B.2
Fluently divide multi-digit numbers using the standard algorithm.

CCSS.MATH.CONTENT.6.NS.B.3
Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

CCSS.MATH.CONTENT.6.NS.B.4
Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*.

Apply and extend previous understandings of numbers to the system of rational numbers.

CCSS.MATH.CONTENT.6.NS.C.5
Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

CCSS.MATH.CONTENT.6.NS.C.6
Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

CCSS.MATH.CONTENT.6.NS.C.6.A
Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., -(-3) = 3, and that 0 is its own opposite.

CCSS.MATH.CONTENT.6.NS.C.6.B
Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.

CCSS.MATH.CONTENT.6.NS.C.6.C
Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

CCSS.MATH.CONTENT.6.NS.C.7
Understand ordering and absolute value of rational numbers.

CCSS.MATH.CONTENT.6.NS.C.7.A
Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret -3 > -7 as a statement that -3 is located to the right of -7 on a number line oriented from left to right*.

CCSS.MATH.CONTENT.6.NS.C.7.B
Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write -3 ^{o}C > -7 ^{o}C to express the fact that -3 ^{o}C is warmer than -7 ^{o}C*.

CCSS.MATH.CONTENT.6.NS.C.7.C
Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of -30 dollars, write |-30| = 30 to describe the size of the debt in dollars*.

CCSS.MATH.CONTENT.6.NS.C.7.D
Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars*.

CCSS.MATH.CONTENT.6.NS.C.8
Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Expressions & Equations

Apply and extend previous understandings of arithmetic to algebraic expressions.

CCSS.MATH.CONTENT.6.EE.A.1
Write and evaluate numerical expressions involving whole-number exponents.

CCSS.MATH.CONTENT.6.EE.A.2
Write, read, and evaluate expressions in which letters stand for numbers.

CCSS.MATH.CONTENT.6.EE.A.2.A
Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 - y*.

CCSS.MATH.CONTENT.6.EE.A.2.B
Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms*.

CCSS.MATH.CONTENT.6.EE.A.2.C
Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6 s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$*.

CCSS.MATH.CONTENT.6.EE.A.3
Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y*.

CCSS.MATH.CONTENT.6.EE.A.4
Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

Reason about and solve one-variable equations and inequalities.

CCSS.MATH.CONTENT.6.EE.B.5
Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use

substitution to determine whether a given number in a specified set makes an equation or inequality true.

CCSS.MATH.CONTENT.6.EE.B.6
Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

CCSS.MATH.CONTENT.6.EE.B.7
Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

CCSS.MATH.CONTENT.6.EE.B.8
Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x <$ c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Represent and analyze quantitative relationships between dependent and independent variables.

CCSS.MATH.CONTENT.6.EE.C.9
Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.

Geometry

Solve real-world and mathematical problems involving area, surface area, and volume.

CCSS.MATH.CONTENT.6.G.A.1
Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

CCSS.MATH.CONTENT.6.G.A.2
Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the

formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

CCSS.MATH.CONTENT.6.G.A.3
Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

CCSS.MATH.CONTENT.6.G.A.4
Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Statistics & Probability

Develop understanding of statistical variability.

CCSS.MATH.CONTENT.6.SP.A.1
Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages*.

CCSS.MATH.CONTENT.6.SP.A.2
Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.

CCSS.MATH.CONTENT.6.SP.A.3
Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Summarize and describe distributions.

CCSS.MATH.CONTENT.6.SP.B.4
Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

CCSS.MATH.CONTENT.6.SP.B.5
Summarize numerical data sets in relation to their context, such as by:

CCSS.MATH.CONTENT.6.SP.B.5.A
Reporting the number of observations.

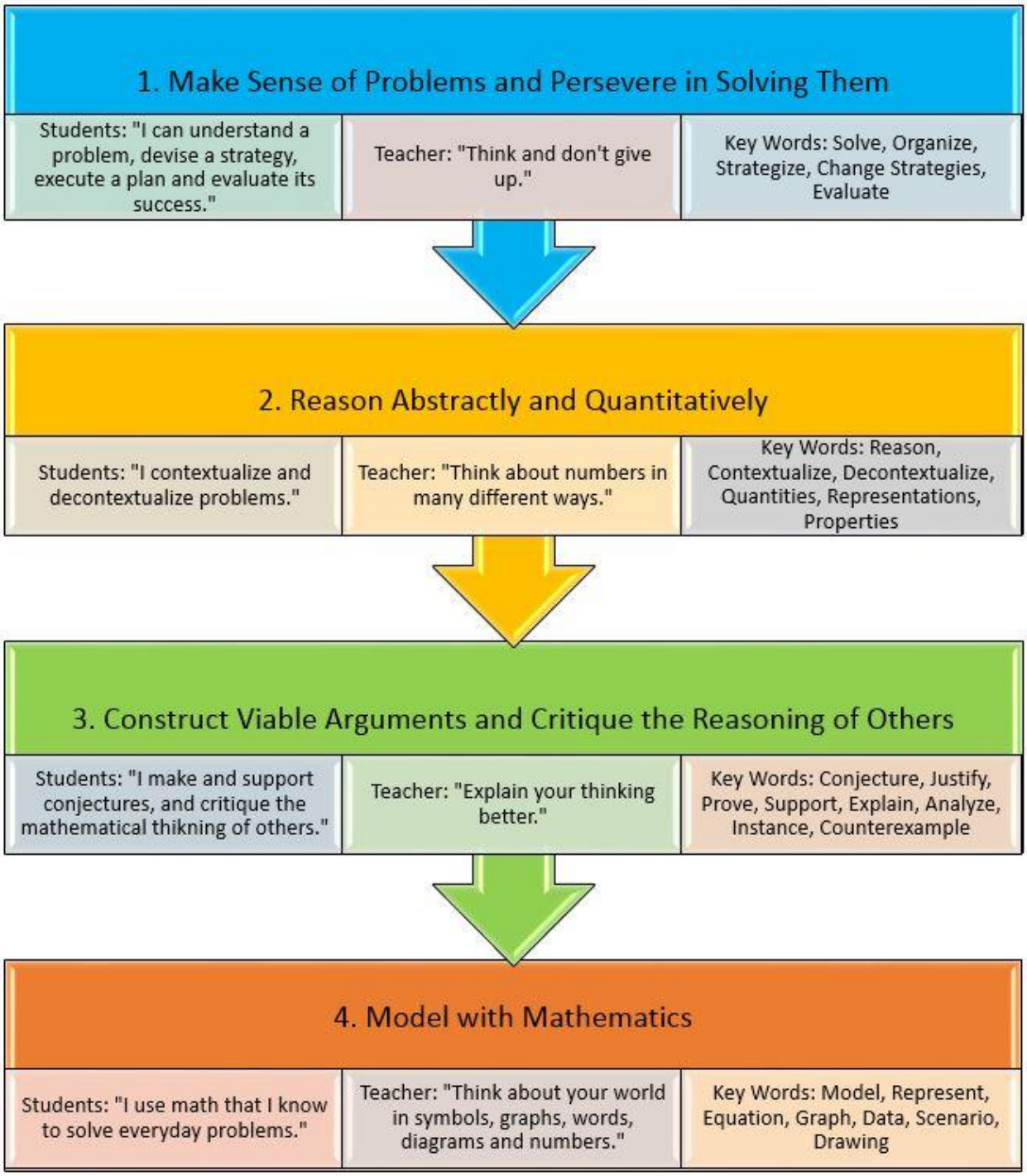
CCSS.MATH.CONTENT.6.SP.B.5.B
Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.

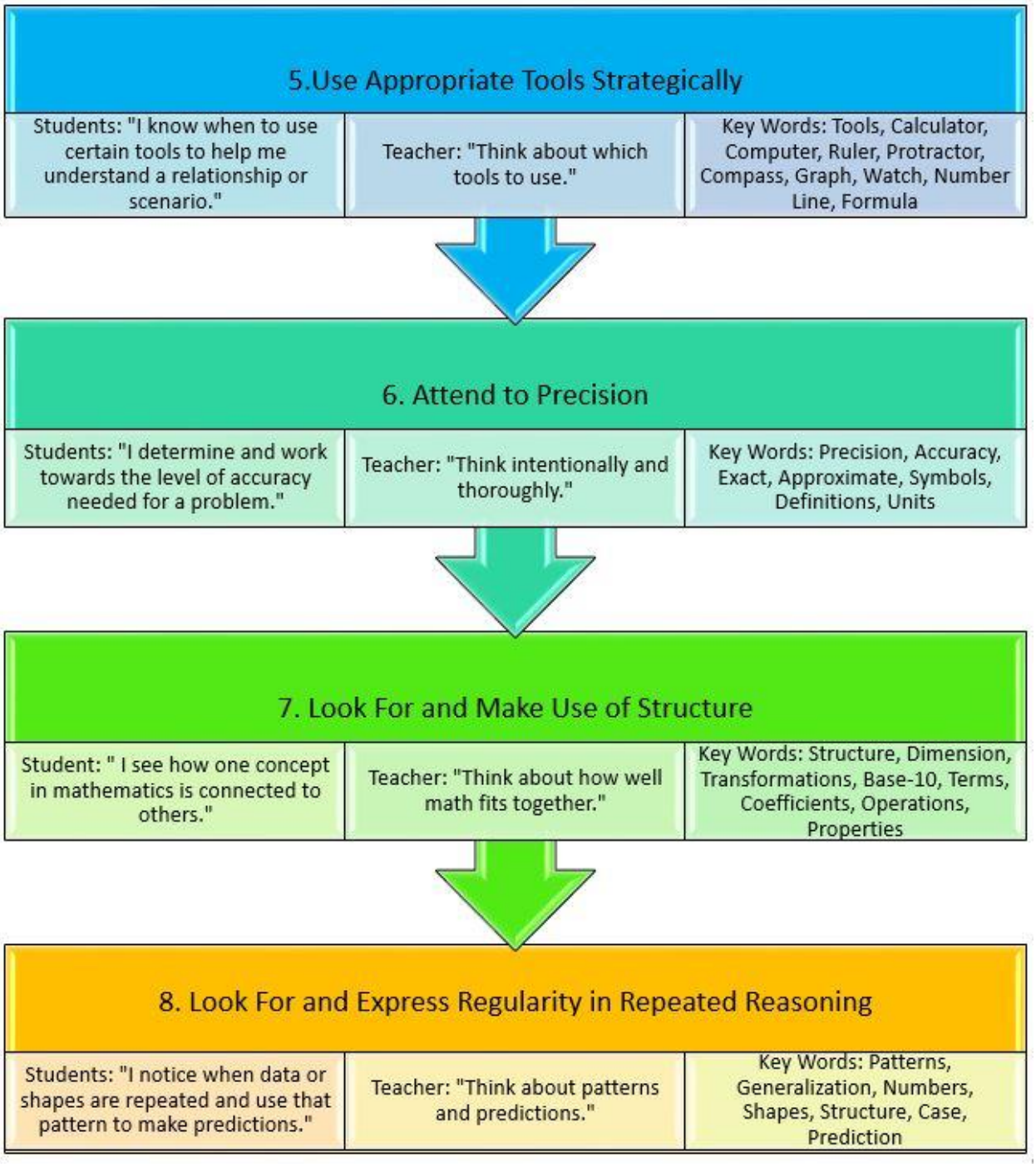
CCSS.MATH.CONTENT.6.SP.B.5.C
Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

CCSS.MATH.CONTENT.6.SP.B.5.D
Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Appendix B

The Mathematical Practices for Teachers



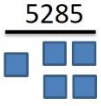

5.Use Appropriate Tools Strategically
Students: "I know when to use certain tools to help me understand a relationship or scenario."
Teacher: "Think about which tools to use."
Key Words: Tools, Calculator, Computer, Ruler, Protractor, Compass, Graph, Watch, Number Line, Formula
6. Attend to Precision
Students: "I determine and work towards the level of accuracy needed for a problem."
Teacher: "Think intentionally and thoroughly."
Key Words: Precision, Accuracy, Exact, Approximate, Symbols, Definitions, Units
7. Look For and Make Use of Structure
Student: " I see how one concept in mathematics is connected to others."
Teacher: "Think about how well math fits together."
Key Words: Structure, Dimension, Transformations, Base-10, Terms, Coefficients, Operations, Properties
8. Look For and Express Regularity in Repeated Reasoning
Students: "I notice when data or shapes are repeated and use that pattern to make predictions."
Teacher: "Think about patterns and predictions."
Key Words: Patterns, Generalization, Numbers, Shapes, Structure, Case, Prediction

Appendix C

Solutions for Partner Activities

<table>
<tr><th colspan="2">Chapter 2</th></tr>
<tr><td colspan="2">Lesson 2.3</td></tr>
<tr><td>Partner Activity 1</td><td>Step 1: Add up both lengths to find the total length
$2\frac{3}{4}+1\frac{1}{2}=\frac{11}{4}+\frac{3}{2}=\frac{11}{4}+\frac{6}{4}=\frac{17}{4}=4\frac{1}{4}$

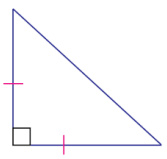
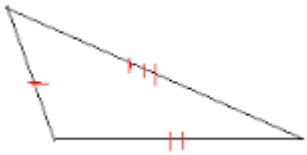
Step 2: Divide the part by the whole
$\frac{Sam}{Total}=\frac{2\frac{3}{4}}{4\frac{1}{4}}=2\frac{3}{4}\div 4\frac{1}{4}=\frac{11}{4}\div\frac{17}{4}=\frac{11}{4}\times\frac{4}{17}=\frac{11}{17}\approx 0.65=65\%\approx\frac{2}{3}$</td></tr>
<tr><td>Partner Activity 2</td><td>$\frac{1\text{ hour}}{84\text{ miles} + 92\text{ miles}}=\frac{1\text{ hour}}{176\text{ miles}}$

$\rightarrow\frac{132\text{ miles needed in one hour}}{176\text{ miles total in one hour}}=\frac{3}{4}\text{hour}=45\text{ minutes}$</td></tr>
<tr><td>Partner Activity 3</td><td> 
→ 5285 ÷ 5 = 1057 = first number
1057 × 4 = 4228 = second number
Check it! 1057 + 4228 = 5285</td></tr>
<tr><td colspan="2">Lesson 2.4</td></tr>
<tr><td>Partner Activity 1</td><td>1. MMXLV = 1000+1000+(50-10)+5 = 2045
2. MDCCLXXXIX = 1000+500+200+50+30+(10-1) = 1789
3. 1993 = 1000+(1000-100)+(100-10)+1+1+1 = MCMXCIII</td></tr>
</table>

	4. 5495 = 5000+(500-100)+(100-10)+5 = **VCDXCV**
Lesson 2.6	
Partner Activity 1	**1A)** 132_{four} **1B)** 200313_{four} **1C)** 100_{four} **2A)** 15 **2B)** 278 **2C)** Undefined
Lesson 2.8	
Partner Activity 1	**1.** 116_{nine} **2.** 1030_{four} **3.** 513_{six} **4.** 21_{three} **5.** 111110_{two}
Chapter 3	
Lesson 2	
Partner Activity 1	Answer: 265
Partner Activity 2	Answers vary
Partner Activity 3	**1.** 408 **2.** 246
	Lesson 4
Partner Activity 1	$5 \div 0 = \text{undefined}$ $0 \div 5 = 0$ $0 \div 0 = \text{indeterminate}$

	We know that $18 \div 6 = 3$ because $3 \times 6 = 18$, therefore. Using a similar approach, we know that $5 \div 0 \neq 0$, since $0 \times 0 \neq 5$
Lesson 3.5	
Partner Activity 1	**1.** $14.47 **2.** $3.65 **3.** $16.32
Partner Activity 2	**1.** 12 X 6 = 72 **2.** 10 X 18 = 180 **3.** 8 X 8 = 64
Partner Activity 3	**1.** 321 **2.** 229 **3.** 15453
Lesson 6	
Partner Activity 1	Answers vary. Samples are below: **1.** Here to Baker, CA **2.** The length of my bedroom **3.** The weight of a 2nd grader
Lesson 7	
Partner Activity 1	**1.** 309.26 **2.** 92.86% **3.** 114.06
Chapter 4	
Lesson 4.2	

Partner Activity 1	**1.** The denominator is VERY MUCH less than the numerator, i.e. $\frac{1}{10000}$ **2.** Both the denominator and the numerator are large numbers and they are very close in distance, i.e. $\frac{999}{1000}$ **3.** The numerator is roughly half of the denominator, i.e. $\frac{15}{31}$ **4.** The numerator is roughly a third of the denominator, i.e. $\frac{21}{60}$
Partner Activity 2	$\frac{3}{8}\to\frac{1}{2},\ \frac{5}{4}\to 1,\ \frac{2}{9}\to 0,\ \frac{4}{7}\to\frac{1}{2},\ \frac{1}{3}\to\frac{1}{2}$
Lesson 4.3	
Partner Activity 1	$\frac{51}{684}+\frac{43}{684}+\frac{738}{684}=\frac{832}{684}$ *or* $\frac{1}{8}+\frac{4}{5}+\frac{1}{9}=\frac{\quad}{360}+\frac{\quad}{360}+\frac{\quad}{360}=$ $\frac{45}{360}+\frac{288}{360}+\frac{40}{360}=\frac{373}{360}$
Partner Activity 2	Division works. The rest do not.
Chapter 5	
Lesson 5.2	
Partner Activity 1	Factors: 1, 2, 3, 5, 6, 10, 15, 30 First four multiples: 30, 60, 90, 120
Partner Activity 2	Primes: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, and 97

Partner Activity 3	0, 1, negative = neither. 2 is the first prime number.
Partner Activity 4	**1.** $85 = 5 \bullet 17$ **2.** $350 = 2 \bullet 5^2 \bullet 7$ **3.** $60 = 2^2 \bullet 3 \bullet 5$
Lesson 5.4	
Partner Activity 1	2 feet; The GCF of (6, 8, 12) = 2
Partner Activity 2	Each packet contains 12 cards; The GCF of (24, 60, 48) = 12
Lesson 5	
Partner Activity 1	120 cookies; The LCM of (15, 24) = 120
Partner Activity 2	**1.** $\frac{1}{12}$ **2.** $\frac{1}{6}$ **3.** 14
Chapter 6	
Lesson 6.2	
Partner Activity 1	**1.**  **2.**  **3.** Does not exist

Partner Activity 2	**1.** A square is a rectangle **2.** Parallelogram **3.** A regular has all same sides and all same angles
Partner Activity 3	**1.** 180 **2.** 540 **3.** 2340 **4.** 360
Partner Activity 4	**1.** 150 **2.** 41
Lesson 6.3	
Partner Activity 1	**1.** Area is base times height, two dimensions **2.** Volume is base times height times width, three dimensions
Partner Activity 2	About 12 $units^2$
Partner Activity 3	52 $units^2$
Partner Activity 4	24 $units^3$
Lesson 6.4	
Partner Activity 1	**1.** $\frac{5}{9}$ ***yards*** **2.** 84480 feet
Lesson 6.5	
Partner Activity 1	**1.** 1200 $inches^2$ **2.** 225 $feet^2$ **3.** 196838560.5 $miles^2$ **4.** 37.68 $inches^3$

Appendix D

Answers to Practice Problems

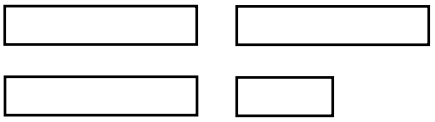
Chapter 2	
Lesson 2.2	**1.** 1063 feet **2.** 5 pounds, 8 ounces **3.** $100 trillion
Lesson 2.3	**1.** There are 5 trikes and 2 bikes **2.** 21 cents **3.** 56 seconds **4.** 7 pieces; yes **5.** $\frac{1}{5}$ are chickens **6.** $198.65
Lesson 2.4	**1.** $\overline{VIII}\,\overline{IV}DCCLIX$ **2.** 2020 **3.** 1982 **4.** $MDCCLXIV$ **5.** $\overline{L}$ **6.** 20089
Lesson 2.5	**1.** The character 4 does not exist in base four **2.** "Five-four-two base six" **3.** 0, 1, 2 10, 11, 12, 20, 21, 22 100, 101, 102 **4.** Answers vary
Lesson 2.6	**1.** **a.** 7565_{nine} **b.** 30_{four} **c.** 10201_{three} **d.** 1001001_{two} **2.** **a.** 46_{ten} **b.** 111_{ten} **c.** 73_{ten} **d.** 123256_{ten}
Lesson 2.7	**1.** Eighty-ones **2.** Six hundred twenty-fives **3.** Six thousand sixty-ones **4.** Thirty eight thousand four hundred sixteens

Lesson 2.8	**1.** 404_{six} **2.** 1011_{two} **3.** 2121_{three} **4.** 104542_{eight}
Chapter 3	
Lesson 3.2	**1.** 50 + 80 + 6 + 1 = 137 **2.** 1000 – 300 + 16 = 716 **3.** 95 + 800 = 895 **4.** 761 – 450 = 760 – 450 + 1 =311
Lesson 3.3	**1.** 7 + 7 + 7 + 7 **2.** 2 + 2 + 2 + 2 + 2 + 2 **3.** 36.6034 **4.** 179.1908
Lesson 3.4	**1.** 7 coworkers; repeat subtraction **2.** 3 pieces of paper; Sharing
Lesson 3.5	**1.** The answers below are precise. Your answers must be close. **a.** 16.5 **b.** 76 **2.** These answers must be precise. **a.** 16.46 **b.** $49.35 **c.** 82 **3.** These answers are precise. Your answer must be close. **a.** $6.48 **b.** $11.36 **c.** $16.39 **4.** Your answers must be exact. **a.** 72 **b.** 72 **c.** Same answers **5.** Your answers must be exact. **a.** 866 **b.** 65322
Lesson 3.6	Answers vary. Here are some sample answers: 1 meter = the height of a short woman 1 inch = distance from nail to join on my thumb 1 foot = length from my elbow to my wrist 1 yard = The height of preschooler
Lesson 3.7	**1.** 309.26 **2.** 92.86% **3.** 114.06

Chapter 4

Lesson 4.2

1. Two possible ways to show $3\frac{1}{2}$:

i. Rectangles: 

ii. Number line: 

2. $\frac{1}{9}, \frac{3}{8}, \frac{2}{5}, \frac{4}{7}, \frac{12}{13}$

Lesson 4.3

1. $\frac{53}{28}$

2. $\frac{11}{28}$

3. $\frac{6}{7}$

4. $\frac{3}{224}$

5. $\frac{257}{30}$ ***or*** $8\frac{17}{30}$

6. $\frac{67}{30}$ ***or*** $2\frac{7}{30}$

7. $\frac{171}{10}$ ***or*** $17\frac{1}{10}$

8. $\frac{162}{95}$ ***or*** $1\frac{67}{95}$

Lesson 4.4

1. Natural, Whole, Integers, Rational and Real
2. Rational and Real
3. Irrational
4. Natural, Whole, Integers, Rational, and Real
5. Irrational
6. Integers, Rational, Real
7. Whole, Integers, Rational and Real
8. Natural, Whole, Integers, Rational and Real
9. 29
10. 147
11. -277

12. -11
13. 276
14. Distributive Property
15. Communitive Property of Multiplication
16. Reflexive Property
17. Transitive Property

Chapter 5

Lesson 5.2

1. 1, 2, 4, 7, 8, 14, 28, **56**
2. 1, 5, 29, **145**
3. 1,2,4,8,16,**32**
4. 50, 100, 150, 200
5. 23, 46, 69, 92
6. 8, 16, 24, 32
7. 23,29,31,37
8. 61,67,71,73,79
9. 127, 131, 137, 139, 149
10. 20, 21, 22, 24, 25, 26, 27, 28, 30, 32, 33, 34, 35, 36, 38, 39, 40
11. 60, 62, 63, 64, 65, 66, 68, 69, 70, 72, 74, 75, 76, 77, 78, 80
12. 120, 121, 122, 123, 124, 125, 126, 128, 129, 130, 132, 133, 134, 135, 136, 138, 140, 141, 142, 143, 144, 145, 146, 147, 148, 150
13. $2^2 \bullet 3^3 \bullet 5$
14. $2^2 \bullet 3 \bullet 5$
15. 5^3

Lesson 5.3

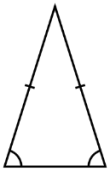
	2	3	4	5	6	8	9	10
67820	X		X	X				X
512	X		X			X		
49								
3463								

Lesson 5.4

1. 10
2. 25
3. 1
4. 30
5. 45

Lesson 5.5

1. 360
2. 75
3. 14280

	4. 1800 **5.** 2025 **6.** $\frac{131}{60}$ **7.** $-\frac{7}{144}$ **8.** $\frac{27}{13}$
Chapter 6	
Lesson 6.2	**1.** 55 **2.** 46 **3.** 121 **4.** 140 **5.** 95 **6.** 90 **7.** Obtuse **8.** Straight **9.** Acute **10.** Right **11.** Acute **12.** Equiangular **13.** Acute scalene **14.** Right scalene **15.**  **16.** Does not exist **17.** Octagon **18.** Heptagon **19.** 360 **20.** 1617 **21.** 1800 **22.** 1260 **23.** Irregular **24.** Regular **25.** Regular **26.** Irregular
Lesson 6.3	**1.** Area is amount inside a two dimensional shape, surface area is the area around a three dimensional shape, volume is the amount of space inside a three dimensional shape **2.** **a.** 2 units^2 **b.** About 2 units^2

	c. About 2.5 units^2 **3.** **a.** 28 units^2 **b.** 24 units^2 **4.** **a.** 7 units^3 **b.** 6 units^3
Lesson 6.4	**1.** 0.00795455 miles or $\frac{7}{880}$ miles **2.** 2.25 yards or $\frac{9}{4}$ yards **3.** 59840 yards **4.** 3276 inches
Lesson 6.5	**1.** 254.3 cm^2 **2.** 28.3 miles^2 **3.** 232.2 cm^2 **4.** 153.9 meters^2 **5.** 25.9 cm^2 **6.** 60.4 miles^2 **7.** 7.3 miles^2 **8.** 2.9 inches^2 **9.** Triangular pyramid **10.** Hexagonal prism **11.** Cylinder **12.** Rectangular pyramid **13.** $59.2\pi\ \boldsymbol{ft}^2$ **14.** $116.4\pi\boldsymbol{in}^2$ **15.** $462\pi\boldsymbol{km}^2$ **16.** 323.5ft^2 **17.** 99 cm^2 **18.** $1152\pi\boldsymbol{m}^3$ **19.** $1.33\pi\boldsymbol{m}^3$ **20.** $1774.67\pi\boldsymbol{km}^3$ **21.** $273.84\pi\ \boldsymbol{ft}^3$ **22.** $27\pi\boldsymbol{mi}^3$ **23.** 168km^3 **24.** $1296\pi\boldsymbol{mi}^3$ **25.** $84\boldsymbol{m}^3$